GETTING THE PICTURE

The role of external representations in simulation-based inquiry learning

Bas Kolloffel

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THE ROLE OF EXTERNAL REPRESENTATIONS IN
SIMULATION-BASED INQUIRY LEARNING

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Thank you all!

Bas Kollöffel,
November 2008

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Chapter 1

Introduction

1 INTRODUCTION

There are many ways to represent information in educational settings: textual descriptions, formulas, photographs, drawings, and so on. As early as the seventeenth century Comenius emphasized that the way in which information is represented is extremely important for effective learning (Schnotz, 2002). Comenius' ideas were followed throughout the centuries, but it lasted until the 1970s before the effects of different representations were studied systematically. Now, after four decades of intensive research, many new insights have been gained and yet understanding the interaction between representations and learning is only beginning to emerge.

1.1 Representations

Palmer (1978) describes a representation as "something that stands for something else. In other words, it is some sort of model of the thing (or things) it represents" (p. 262). A distinction can be made between external representations and mental (or internal) representations, referring to respectively outside or inside the human mind. Examples of external representations are pictures, diagrams, texts, graphs, tables, and symbols. Internal representations on the other hand, are knowledge and structures in human memory, like mental models, propositions, and schemata. The main starting point of the studies presented in this dissertation is on external representations and what is studied is their influence on learning results and thus indirectly on internal representations. Instances where the term "representation" is used without reference to external or internal, can be considered as referring to external representations.

1.1.1 Types of external representations

The number of representation types is nearly countless and so is the number of classifications. For example, in the field of semiotics, Peirce (1998) suggested to classify representations on the basis of the extent to which they resemble the object they represent ranging from representations that (largely) resemble the object (e.g., photographs) to representations that do not resemble the represented object at all and can even represent many different objects. Words are examples of the latter. A word may have several meanings but the context in which it is used, constrains the possible interpretations of the meaning.

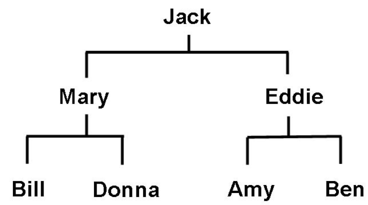
Representations can also be classified on the basis of their attributes. For example, Lohse et al. (1994) had subjects classify 60 visual representations. From the descriptions provided by the subjects, it was found that subjects used ten dimensions (e.g., spatial-nonspatial, temporal-nontemporal, concrete-abstract) to classify different types of visual representations, from which 11 distinct categories of visual representations were derived: graphs, tables, graphical tables, time charts, networks, structure diagrams, process diagrams, maps, cartograms, icons, and pictures. Lohse et al. remark that this list is not necessarily exhaustive and furthermore subdivisions within categories have been left out of consideration.

Another approach can be found in the field of cognitive sciences, where external representations are classified on the basis of how people process information. Usually two categories are distinguished: nonverbal (e.g., pictures, diagrams) or verbal representations (e.g., natural and arithmetical languages) (Klein, 2003; Paivio, 1990). Leading views in cognitive science, for example dual coding theory (Paivio, 1990), dual channel assumption (Mayer, 2003), and

Baddeley's (1997) model of working memory that includes a visuospatial sketchpad and phonological loop, postulate that representations are processed, encoded, and stored by two different cognitive systems, one for nonverbal information and one for verbal information. In this field, research efforts regarding representations often focus on determining if and when representations are efficient for problem solving, learning, and understanding. Research in the 1970s and 1980s established that the efficiency of representations for reasoning and problem solving depends on how representations facilitate search, recognition, and inferential processes, that means how they summarize or highlight essential information, make relations among elements explicit, and organize information into coherent structures (Koedinger & Anderson, 1990; Larkin & Simon, 1987; Levin, 1981; Levin, Anglin, & Carney, 1987; Levin & Mayer, 1993). For example, Figure 1-1 displays two representations that both can be used to find the answer to the question "Is Amy Bill's cousin?".

Jack is Mary's parent
Jack is Eddie's parent
Mary is Bill's parent
Mary is Donna's parent
Eddie is Amy's parent
Eddie is Ben's parent

(a)



(b)

Figure 1-1. Textual and diagrammatic representations of kinship (after Winn, 1993)

Although Figure 1-1a and Figure 1-1b contain equivalent information, Figure 1-1b is generally found to be computationally less demanding to search, recognize, and infer relations. The issue of representational efficiency is particularly relevant for learning and instruction.

1.2 Representations and learning

Representations in learning settings can have many forms and functions. A good match between the type of representation and learning demands can greatly support learning and contribute to enhanced levels of performance and understanding (Ainsworth, 2006; Greeno & Hall, 1997). Over the last decades many research efforts have been invested in studying the effects of representations on learning. The empirical findings are often unequivocal or contradictory and it is increasingly recognized that it is not so much a matter of finding *if* a representation is effective for learning, but rather *when*, under which conditions, it is effective.

1.2.1 Effectiveness of representations in learning settings

From the 1990s onwards, researchers and theorists in the field of learning and instruction increasingly emphasized that the effectiveness of a representation depends on a complex interaction between the nature and goal of the task and the student's familiarity with both the representation and the domain (Ainsworth, 2006; Scaife & Rogers, 1996; Tabachneck-Schijf, Leonardo, & Simon, 1997).

Depending on the nature or goal of the task, one representation can be more appropriate than another. For example, it may be easier to explain the flow of blood through heart, lungs, and body visually, whereas verbal descriptions can be particular effective for representing concepts (e.g., "cognition") (Schnotz, 2002).

The effectiveness of representations also depends on the student's familiarity with the representation. First, students need to understand the form of the representation, how it encodes information, and how it relates to the domain it represents (Ainsworth, 2006). Second, some representational formats require time and practice before they have beneficial effects on learning. For example, Leung, Low, and Sweller (1997) studied learning from equations as compared to words. They found that for a short acquisition time a verbal format led to superior results, but with more practice time available this trend was reversed in favor of the equation format.

Finally, with increasing domain understanding, students become less dependent on the type of representation and become more able to switch between different types of representations (Tabachneck-Schijf et al., 1997).

1.2.2 Pedagogical functions of representations

Often, more than one type of representation appears to qualify for being used in a learning situation. An informed choice for one type of representation or another to support learning can be made on several grounds (Ainsworth, 2006; Scaife & Rogers, 1996). For example, a representation can be used because it causes less cognitive load compared to other representations. For example, 73 x 27 and LXXIII.XXVII are two ways of representing a multiplication problem. Both representations have the same formal structure, yet for most people the Arabic numerals are easier to use than Roman numerals, simply because they are used to solve multiplication problems using Arabic numerals (Zhang & Norman, 1994). Representations can also be selected on the basis of the extent to which they constrain the kinds of inferences that can be made about the represented information (Stenning & Oberlander, 1995). In this case, the emphasis is on how much a representation promotes clarity and/or reduces ambiguity compared to another representation. For example, compared to an indeterminate description like "The knife is to the right of the plate; the fork is to the left of the knife.", a picture or diagram unambiguously expresses the position of the fork (after Mani & Johnson-Laird, 1982). By combining types of representations, the learning process can be supported in more than one way.

1.2.3 Multiple representations

Combining two or more representational formats into what are called *multiple representations* (e.g., van Someren, Reimann, Boshuizen, & de Jong, 1998) is assumed to have some additional effects on knowledge construction processes (Ainsworth, 1999, 2006; Seufert, 2003). First, different formats can complement each other; for example, combining an equation and a diagram can be helpful in focusing the students' attention on not only operational aspects but also conceptual aspects of the domain. Second, one representation can constrain the interpretation of the other. For example, when an arithmetical representation such as an equation is accompanied by a textual representation, the latter might help students to better understand the equation. Third, students' integration of information from different representations is thought to support the construction of deeper understanding (Ainsworth, 1999, 2006; van der Meij & de Jong, 2006).

However, combining representations is not always beneficial for learning. It can interfere with cognitive processing (e.g., split-attention effects), and multiple representations may contain redundant information, which is assumed to increase cognitive load (e.g., Leung et al., 1997).

1.2.4 Student-constructed representations

External representations can be presented to students, but students can also construct representations themselves. Cox (1999) argues that the process of constructing a representation helps students to improve their knowledge, because the interaction between their internal representation and the external representation they construct, can make them aware of gaps in their internal representations they had not noticed before. Examples of activities in which students construct an external representation are: writing a summary (Foos, 1995; Hidi & Anderson, 1986), creating a drawing (Van Meter, Aleksic, Schwartz, & Garner, 2006; Van Meter & Garner, 2005), building a runnable computer model (Löhner, Van Joolingen, & Savelsbergh, 2003; Manlove, Lazonder, & de Jong, 2006; van Joolingen, de Jong, Lazonder, Savelsbergh, & Manlove, 2005), or constructing a concept map (Nesbit & Adesope, 2006; Novak, 1990, 2002). Constructing representations can have different purposes. For example, for students with no or little domain knowledge it can help them build their knowledge; for students with advanced levels of domain knowledge, constructing a representation can serve as an aid to accessing information stored in long term memory and as a summary of their processing, which decreases working memory load and thus helps them to concentrate on reasoning (Tabachneck-Schijf et al., 1997).

1.2.5 Constructing representations in collaborative learning settings

Also in collaborative learning, external representations and their format may play a crucial role in determining the effectiveness of the learning environment. In addition to beneficial effects of the construction of a representation of the domain per se, in collaborative learning an external representation may form the pivot around which students share and discuss knowledge (e.g., Greeno & Hall, 1997; Lesh & Lamon, 1992). Again format may play a role as well. Electronic tools designed to enable students to construct, discuss, and share external representations, often referred to as *representational tools* (e.g., Suthers & Hundhausen, 2003; Toth, Suthers, & Lesgold, 2002), can have different formats. The choice of representational format for these tools is often not based on systematic comparisons of the effects of representations on collaborative learning. Yet, several studies established that the focus of students' discourse and collaborative activities were influenced by the format in which students had to construct a representation (e.g., Suthers & Hundhausen, 2003; van Drie, van Boxtel, Jaspers, & Kanselaar, 2005).

In the previous sections it has been outlined that external representations can play various roles in the learning process. They can help students to select, organize, and integrate information into meaningful and coherent internal representations, being it by communicating information to students in clear and understandable ways, or by serving as a means through which students express, refine, and communicate their understanding. Unfortunately, a clear-cut recipe for which representational format to use when does not exist. Moreover, some researchers argue that the effects of representations found in one domain cannot readily be generalized to other domains (Cheng, Lowe, & Scaife, 2001; Scaife & Rogers, 1996; Zhang, 1997). The studies described in this dissertation focus on the effects of representations in the domain of combinatorics and probability theory, which is a subdomain of mathematics.

1.3 Representations in mathematics

The domain of mathematics that is used in the studies presented hereafter is hard to grasp for many students. A general reason for the problems students experience in domains like mathematics and science is that they often tend to focus on superficial details rather than on understanding the principles and rules underlying a domain (Chi, Feltovich, & Glaser, 1981; de Jong & Ferguson-Hessler, 1986; Reiser, 2004). Mathematics and science problems require students to go beyond the superficial details in order to recognize the concepts and structures that underlie the problem and to decide which operations need to be performed to solve it (e.g., Fuchs et al., 2004). This is particularly true for the domain of combinatorics and probability theory, where problem solving is very dependent on the correct classification of the problem (Lipson, Kokonis, & Francis, 2003). Complicating factor is that combinatorial and probability problems and ideas often appear to conflict with students' experiences and how they view the world (Garfield & Ahlgren, 1988; Kapadia, 1985). Even high-school teachers of statistics have great difficulty correctly conceiving and solving probability problems (Liu & Thompson, 2007). The conflicts arise because probabilities do not always fit people's conceptions and intuitions (Batanero & Sanchez, 2005; Fischbein, 1975; Greer, 2001). An example of a misconception is the gambler's fallacy, that is, the belief that the outcome of a random event can be affected by (and therefore predicted from) the outcomes of previous events. Part of the problems students experience in the domain of mathematics relate to how the domain is represented.

1.3.1 Representations in the domain of mathematics

Some of the students' difficulties with mathematics are caused by the abstract and formal nature of arithmetical representations which do not explicitly show the underlying principles or concepts. Most students tend to view mathematical symbols (e.g., multiplication signs) purely as indicators of which operations need to be performed on adjacent numbers, rather than reflections of principles and concepts underlying these procedures (Atkinson, Catrambone, & Merrill, 2003; Cheng, 1999; Greenes, 1995; Nathan, Kintsch, & Young, 1992; Niemi, 1996; Ohlsson & Rees, 1991). Therefore, they easily lose sight of the meaning of their actions. In this case, processing formal notations becomes an end in itself (Cheng, 1999). Learning arithmetical procedures without conceptual understanding tends to be error prone, easily forgotten, and not readily transferable (Ohlsson & Rees, 1991). Furthermore, the formal, abstract way in which subject matter is represented makes it hard for students to relate the subject matter to everyday life experiences. Fuson, Kalchman, and Bransford (2005) argue that the knowledge students bring into the classroom is often put aside in mathematics instruction and replaced by procedures that disconnect problem solving from meaning making.

1.3.2 External representations in combinatorics and probability theory

There are several ways of representing information in combinatorics and probability theory. Three examples will be shown that are all based on the following problem:

Your bank distributes a random four-digit code as a personal identification number (PIN) for its credit card. What is the probability that a thieve finding the card and trying to get money with it will guess the correct code in one go, and will be able to plunder your account?

One of the most common ways to represent the steps towards solving this type of problem is by means of a tree diagram (see Figure 1-2).

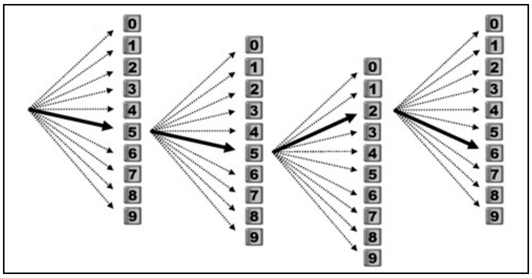


Figure 1-2. Tree diagram representing solution PIN-code problem

Tree diagrams are considered a powerful tool for teaching combinatorics and probability theory (e.g., Fischbein, 1987; Greer, 2001). They are especially effective in assessing the probability of various options (Fischbein, 1987; Halpern, 1989). Comprehension of tree diagrams requires some level of familiarity with conventions specifying the meaning of the diagram (Cobb, 1989; Fischbein, 1987). A second way to represent the PIN-code problem is by using an arithmetical representation (see Figure 1-3).

$$p(\text{PIN}=5526) = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$$

Figure 1-3. Equation representing PIN-code problem

This representation is informationally equivalent with the tree diagram, although recognizing the parallels may strongly depend on the student's knowledge of the meaning of arithmetical representations. One needs to know, for example, the conceptual meaning of the multiplication sign. Most students will interpret it as a calculation rather than as a representation of a principle or concept. A textual way of representing the PIN-code problem is displayed in Figure 1-4. The use of natural language facilitates relating information in the text to everyday experiences and situations. On the other hand, problems with text comprehension may hamper problem solving performance (Koedinger & Nathan, 2004; Lewis & Mayer, 1987; Nathan et al., 1992).

When selecting the first digit of a PIN-code, one can choose from ten digits: 0, 1, 2, up to 9. The chance that 5 will be selected as the first digit is equal to one out of ten. When selecting the second digit of the PIN-code, one can choose from ten digits again, because the digit that was selected the first time, can be selected again.
The chance that 5 is selected as second digit of the code is therefore equal to one out of ten possible digits.
The chance that 2 is selected as the third digit of the code is also equal to one out of then possible digits, and so is the chance that 6 is selected as fourth digit.

Figure 1-4. Text representing PIN-code problem

1.3.3 Student-constructed representations in combinatorics and probability theory

With regard to student-constructed representations of information in the domain of combinatorics and probability theory, it has been found that students avoid using conventional ways of representing the probability of events (i.e., using ratios or odds, or formal numerical probabilities) and prefer to use alternative forms of representation, ranging from textual statements to conventional numerical representations (Tarr & Lannin, 2005). This finding indicates that not all formats may be equally suitable for students trying to express their knowledge. The aim of the research presented in this dissertation is to examine the effects of external representations on learning in the domain of combinatorics and probability theory. The focus will be both on external representations *presented to* and external representations *constructed by* students. The questions that will guide the studies will be specified in the next section.

2 RESEARCH QUESTIONS

The overall research question of the project is: how does representational format facilitate knowledge construction processes and how does this influence learning?

The overall research question will be investigated in a step-by-step way. Three studies will be conducted that all use basically the same learning environment on combinatorics and probability theory (see also Sections 3.1 and 3.3).

Study 1: Does representational format influence learning combinatorics?

The first study investigates which representations help students best to acquire domain knowledge. In this first study students work individually. Five conditions are compared to each other: three conditions each using a single external representational format (Diagrammatical, Arithmetical, or Textual), and two conditions using combinations of single representational formats (Textual + Arithmetical or Diagram + Arithmetical). Following Larkin and Simon's (1987) baseline for drawing valid conclusions from comparisons of representations, the representations were kept as informationally equivalent as possible. The effects of representational formats are evaluated in terms of effects on knowledge construction and efficiency. This study is presented in Chapter 2.

Study 2: What are the effects and perceived affordances of the format of representational tools?

The second research question relates to how students express the contents of the domain and how they explain this to other students. Now students, while working in the learning environment, are able to express and present their ideas to a fellow student. The learning environment offered contains the representational format that in Study 1 was found to be the most optimal. Students receive an electronic representational tool that has either a conceptual, arithmetical, or textual format. Students are asked to construct a representation of the domain that explains the domain to another fictitious student. The effects of representational formats are evaluated in terms of effects on knowledge construction. This study is presented in Chapter 3.

Study 3: What are the effects and perceived affordances of the format of representational tools in a collaborative learning setting?

In the third and final study the collaboration aspect is introduced. Basically, the set-up is the same as in the second study, but now two students are learning collaboratively. The representational tool is now a shared representational tool that again is “pre-structured” in either a conceptual, arithmetical, or textual way. The effects of representational formats are evaluated in terms of effects on knowledge construction. This study is presented in Chapter 4.

3 RESEARCH FRAMEWORK

The studies described in this dissertation were part of a research programme (“aandachtsgebied”) coined “Learning Environments, MultiMedia, and Affordances” (LEMMA). Four universities participated in LEMMA to examine the relations between external representational codes (pictorial, arithmetical, and textual), learning processes, and learning outcomes. The aim of LEMMA was to determine the most optimal representations to support the various learning processes, and to ultimately offer practical guidelines for designing learning environments. All LEMMA studies used the same domain, combinatorics and probability theory. Furthermore, much of the materials like examples and problems, introductory text, and measurement instruments were developed cooperatively, and used by all participants. The main difference between the projects within the LEMMA framework concerned the instructional approaches, which are: *hypermedia learning* (Institut für Wissensmedien-Knowledge Media Research Center (IWM/KMRC), Tübingen Germany), *observational learning* (Open University Netherlands), and *self-explanation based learning* (University of Freiburg, Germany). The instructional approach used in the studies reported in this dissertation is *simulation-based inquiry learning*.

3.1 Domain

The essence of combinatorics is determining how many different combinations can be made with a certain set or subset of elements. In order to determine the number of possible combinations, one also needs to know 1) whether elements may occur repeatedly in a combination (replacement) and 2) whether the order of elements in a combination is of interest (order). On basis of these two criteria, four so-called problem categories can be distinguished (see Figure 1-5).

		ORDER IMPORTANT?	
		Yes	**No**
REPLACEMENT?	**No**	Category 1: No replacement; Order important	Category 2: No replacement; Order not important
	Yes	Category 3: Replacement; Order important	Category 4: Replacement; Order not important

Figure 1-5. Problem categories within the domain of combinatorics

As part of the LEMMA cooperation, for each category a cover story was created. A cover story is a short story about a realistic situation and/or problem exemplifying the problem category in question. The PIN-code problem presented on page 7 is an example of the cover story used for the category 3, "replacement; order important". The cover stories of the other three categories are displayed in Appendix 1.

3.2 Measures (pre-test and post-test)

Two knowledge tests were developed for the LEMMA framework: a pre-test and a post-test. The pre-test aimed at measuring (possible differences in) the prior knowledge of the participants. The post-test aimed at measuring the completeness of students' schemas related to this domain. Sweller (1989, p. 458) defined a schema as "...a cognitive construct that permits problem solvers to recognize problems as belonging to a particular category requiring particular moves for solution". A complete schema therefore rests on three pillars: conceptual knowledge, procedural knowledge, and situational knowledge. *Conceptual knowledge* is "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson, Siegler, & Alibali, 2001, p. 364). Conceptual knowledge develops by establishing relationships between pieces of information or between existing knowledge and new information. An example of a post-test item aiming at measuring conceptual knowledge is provided in Figure 1-6.

You visit the horse races. It's your first time and you have no idea which horses are good. You predict which horse will win and you make a bet on that. The race starts and there is one horse that wins with a large lead head. All the other horses are still in the race and the differences are minimal. You can still make a bet on who will become second. Do you have a bigger chance to correctly predict the number two compared to the chance you had to predict the number one?

a. Yes, the chance increases with my second bet
b. No, the chance is the same for my second bet as it was for my first bet
c. No, my chance decreases with my second bet
d. There is no systematic relation between my first and my second bet

Figure 1-6. Post-test item measuring conceptual knowledge

Some items were intended to measure intuitive conceptual knowledge (see Figure 1-7 for an example). Items measuring conceptual knowledge and intuitive conceptual knowledge differed in three respects (Eysink et al., submitted): first, the situation described in the problem statement regarding the intuitive items was the same for each item and was presented prior to the items instead of being presented with each separate item; second, the intuitive items offered two alternatives instead of four; finally, students were asked to answer the intuitive items as quickly as possible, as intuitive knowledge is characterized by a quick perception of meaningful situations (Swaak & de Jong, 1996).

(Answer the following question(s) as quickly as possible)

There are a number of marbles in a bowl. Each marble has a different color. You will pick at random (e.g., blindfolded) a number of marbles from the bowl, but before you do you predict which colors you will pick.

The chance your prediction proves to be correct is higher in case of:

a. No replacement; order not important
b. Replacement; order important

Figure 1-7. Post-test item measuring intuitive conceptual knowledge

Procedural knowledge is "the ability to execute action sequences to solve problems" (Rittle-Johnson et al., 2001, p.346). An example of a post-test item aiming at measuring procedural knowledge is provided in Figure 1-8.

You and your friend participate in a lottery. The lottery draws a first and a second prize out of 100 different numbers. You cannot win more than one prize per lot. You both have 1 lot and you bet with your friend that you will win the first prize and he will win the second prize. What is the probability that you win the bet?

Figure 1-8. Post-test item measuring procedural knowledge

Situational knowledge (de Jong & Ferguson-Hessler, 1996) enables students to analyze, identify, and classify a problem, to recognize the concepts that underlie the problem, and to decide which operations need to be performed to solve the problem. An example of a post-test item measuring this type of knowledge is displayed in Figure 1-9.

You had a party and bought balloons in different colors. Now the party is over, you can pin the balloons. You will do this blindfolded and you predict that you pin a red one first, then a yellow one and finally a blue one. What is the characterization of this problem?

a. order important; replacement
b. order important; no replacement
c. order not important; replacement
d. order not important; no replacement?

Figure 1-9. Post-test item measuring situational knowledge

The correct answers to the items presented in Figure 1-6, Figure 1-7, Figure 1-8, and Figure 1-9, are respectively: answer A; answer A; (1/100)*(1/99)=1/9900; and answer B.

The original LEMMA post-test consisted of 44 items: 25 conceptual knowledge items, 14 procedural knowledge items, and 5 situational knowledge items. After Study 1, post-test data from all LEMMA-partners were collected and analyzed. Item-analyses were carried out in an iterative fashion, with the following criteria: baseline test reliability is Cronbach's Alpha .70 and a baseline of .30 for the Corrected Item-Total Correlation. The iterative item-analyses showed that 18 items could be deleted from the test, almost without any consequences for the reliability (Cronbach's Alpha was .83 before deletion and .81 after deletion). Of the 25 items used for measuring conceptual knowledge, 13 were deleted because of poor corrected item-total correlations. Deletion of these items even slightly improved reliability (from .74 to .76). Of the 14 items for measuring procedural knowledge, 4 were deleted (reliability was .73 before deletion, and .74 after deletion). With regard to the five items measuring situational knowledge, one item showed a poor corrected item-total correlation (.24). Leaving out this item improved reliability from .65 to .67. The revised post-test therefore consisted of 26 items. This version of the post-test was used in Study 2 and 3.

3.3 Instruction

The instructional approach used in the studies reported in this dissertation is simulation-based inquiry learning (de Jong, 2005, 2006). In inquiry learning, the focus of instruction is primary on the induction of concepts and principles of a domain (Swaak & de Jong, 1996). Students inquire the properties of the given domain (de Jong & van Joolingen, 1998; van Joolingen, 1993; van Joolingen & de Jong, 1997).

Computer-based simulation is a technology that is particularly suited for inquiry learning. Computer-based simulations contain a model of a system or a process. By manipulating the input variables and observing the resulting changes in output values the student is enabled to induce the concepts and principles underlying the model (de Jong & van Joolingen, 1998).

Although active and meaningful learning are viewed as main characteristics of inquiry learning (Svinicki, 1998), the relation between activity and meaningfulness in learning should be considered with care. Mayer argues that meaningful learning may not simply be the result of behavioral activity per se. He suggests that only specific cognitive activities (e.g. selecting, organizing, and integrating knowledge) may promote meaningful learning (Mayer, 2002, 2004). In order to have students deploying the required and appropriate cognitive activities and to prevent them from floundering, students need some level of support. Leaving students to their own devices is not a very effective and efficient way of learning, or, as Mayer puts it: "a formula for educational disaster" (2004, p.17). Integrating supportive cognitive tools in the learning environment can help students to learn more effectively (de Jong, 2006). Another way to support knowledge construction processes in inquiry learning is to let students work collaboratively (Gijlers & de Jong, 2005).

The learning environments used in the current study were created with SIMQUEST authoring software (van Joolingen & de Jong, 2003). The learning environments consisted of several sections. For each of the four problem categories a section was developed. These four sections all had the same structure. First, the cover story exemplifying the problem category dealt with in the section was introduced (see Appendix 1). Then, the students were presented with a series of assignments (both open-ended and multiple-choice items), all based on the cover story of that particular section. These assignments involved determining which problem category matched the given cover story; calculating a probability in a given situation; inquiring the structure of problem solving procedures; inquiring the relations between variables within the problem category; and inquiring the relation with another problem category.

A fifth section was added aiming at integrating the four problem categories. In this section the "Bicycle" cover story was used, which was designed in such a way that it could be applied to all problem categories (see Appendix 1). In this section, the students were presented with four category classification assignments and three assignments in which they were presented with hypotheses they had to inquire.

Most of the assignments in each of the five sections were accompanied by simulations based on the cover story that could be used to explore the relations between variables within the problem category in question (see Figure 1-10).

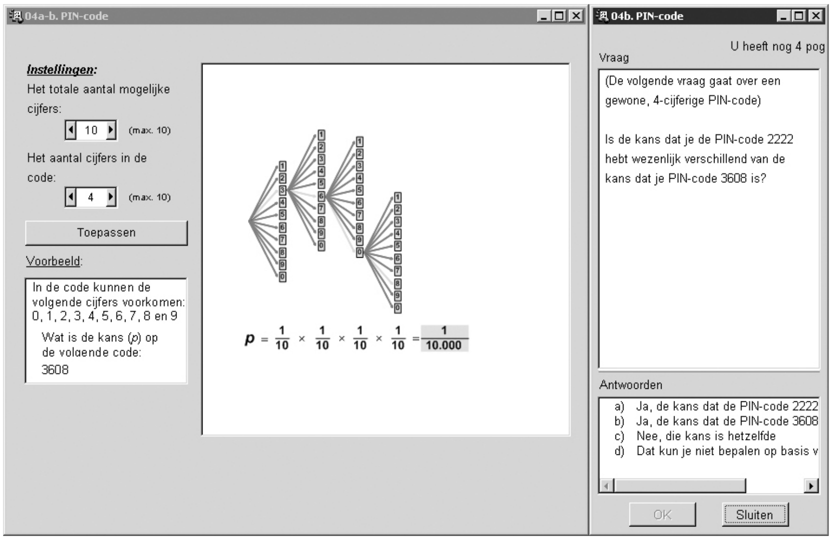


Figure 1-10. Screen dump simulation and assignment (right) about PIN-code problem

In the simulations students could manipulate variables and observe the effects of their manipulations on other variables. In the case of the multiple-choice items, the students received feedback from the system about the correctness of their answer. If the answer was wrong, the system offered hints about what was wrong with the answer. Students then had the opportunity to select another answer. In the case of the open-ended assignments, students received the correct answer after completing the assignment.

4 DISSERTATION OVERVIEW

As discussed in Section 2, the overall research question was investigated in a step-by-step way. Three empirical studies were conducted. The following three chapters (Chapter 2 through 4) will each present one of these studies. Finally, in Chapter 5 the results and conclusions of all studies are reviewed and discussed. Theoretical and practical implications will be discussed and the outcomes will be translated into practical guidelines for instructional designers and teachers.

Chapter 2

Effects of format on learning from a computer-simulation[1]

Abstract

The current study investigated the effects of different external representational formats on learning combinatorics and probability theory in an inquiry based learning environment. Five conditions were compared in a pre-test post-test design: three conditions each using a single external representational format (Diagram, Arithmetic, or Text), and two conditions using multiple representations (Text + Arithmetic or Diagram + Arithmetic). The major finding of the study is that a format that combines text and arithmetics was most beneficial for learning, in particular with regard to procedural knowledge, that is the ability to execute action sequences to solve problems. Diagrams were found to negatively affect learning and to increase cognitive load. Combining diagrams with arithmetical representations reduced cognitive load, but did not improve learning outcomes.

[1] This chapter is adapted from Kolloffel, B., Eysink, T. H. S., de Jong, T., & Wilhelm, P. (in press). The effects of representational format on learning combinatorics from an interactive computer-simulation. *Instructional Science.*

1 INTRODUCTION

The format of external representations (symbols, diagrams, et cetera) is known to play a critical role in learning and understanding (Ainsworth & Loizou, 2003; Cheng, 1999; Zhang, 1997). External representations are usually classified into two categories: nonverbal (e.g., diagrams) and verbal representations (e.g., natural and arithmetical languages) (Klein, 2003; Paivio, 1990). In general, diagrams are associated with superior performance compared to textual material (Goolkasian, 2000; Marcus, Cooper, & Sweller, 1996). Diagrams are considered to be most useful as an aid to understanding when materials are complex or difficult to understand (Levin, 1981). In a meta-analysis Levin, Anglin, and Carney (1987) found that diagrams are most effective in enhancing learning if they organize events into a coherent structure, clarify complex and abstract concepts, or assist students in recalling important information. Diagrams are effective because they make relations among elements explicit and make information more concise by summarizing or highlighting what is essential (Levin & Mayer, 1993).

Many researchers have observed that, despite a vast body of research on the facilitative effects of diagrams, not much is known about the cognitive processes underlying these effects (Cheng, 1999; Glenberg & Langston, 1992; Goolkasian, 2000; Scaife & Rogers, 1996; Zhang, 1997). In addition, not much is known about the behavioral aspects of representations. These are important because modern instructional computer technology increasingly provides students with interactive representations. In simulation-based instruction, for example, students can manipulate representations in order to explore the concepts and principles underlying the domain (de Jong & van Joolingen, 1998). Cheng (1999) argues that the active manipulation of representations makes underlying domain relations more accessible compared to static representations. Rogers (1999) adds that interactive representations can reduce the amount of “low-level” cognitive activities (e.g., drawing and redrawing) normally required when learning, and in turn allow students to devote their cognitive resources to more “high-level” cognitive tasks, such as exploring more of the problem space.

2 REPRESENTATIONAL FORMATS

The properties of a representation are assumed to influence which information is attended to and how people tend to organize, interpret, and remember the information presented. Larkin and Simon (1987) state that the value of representations depends on two factors: informational and computational efficiency. *Informational efficiency* refers to how representations organize information into data structures. *Computational efficiency* refers to the ease and rapidity with which inferences can be drawn from a representation. Even when a diagram and a text are informationally equivalent, meaning that all of the information in one representation can be inferred from the other and vice versa, the diagram is often more effective because inferences can be drawn more quickly and easily from diagrams (Koedinger & Anderson, 1990; Larkin & Simon, 1987).

We will use an example relevant to the understanding of combinatorics, the problem of a thief guessing the four-digit PIN-code (5526) of a credit card he has just stolen, to illustrate different ways of organizing information into data structures. Guessing the PIN-code can be conceived as a process of four interrelated steps: the first step is selecting the first digit of the code. The thief has 10 options (0 through 9) of which one is correct. The second step is selecting the second digit, and again there are 10 options of which one is correct. This is also true for the third step (selecting the third digit) and the fourth step. These four steps can be represented in different ways, such as by means of diagrams, texts, or arithmetic.

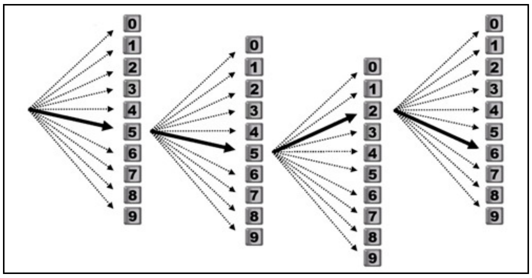


Figure 2-1. Tree diagram representing solution PIN-code problem

In Figure 2-1 the PIN-code problem is represented diagrammatically as a tree diagram. In pictures or diagrams, information is indexed by a two-dimensional location in a plane, explicitly preserving information about topological and structural relations (Larkin & Simon, 1987). Comprehension of diagrams involves the establishment of some conventions specifying the meaning of the diagram (Cobb, 1989; Fischbein, 1987). Tree diagrams could also preserve sequential relations, specifying the available options at each step. Tree diagrams are considered a powerful tool for teaching combinatorics and probability theory (e.g., Fischbein, 1987; Greer, 2001). They are especially effective in assessing the probability of various options (Fischbein, 1987; Halpern, 1989).

When selecting the first digit of a PIN-code, one can choose from ten digits: 0, 1, 2, up to 9. The chance that 5 will be selected as the first digit is equal to one out of ten. When selecting the second digit of the PIN-code, one can choose from ten digits again, because the digit that was selected the first time, can be selected again.
The chance that 5 is selected as second digit of the code is therefore equal to one out of ten possible digits.
The chance that 2 is selected as the third digit of the code is also equal to one out of then possible digits, and so is the chance that 6 is selected as fourth digit.

Figure 2-2. Text representing PIN-code problem

In Figure 2-2 the PIN-code problem is represented textually. The use of natural language facilitates the relating of information in the text to everyday experiences and situations. On the other hand, problems with text comprehension may hamper problem solving performance (Koedinger & Nathan, 2004; Lewis & Mayer, 1987; Nathan et al., 1992). What distinguishes a text from a mere set of sentences is that a text is cohesive. Sentences in a text build upon and refer to one another. Textual representations emphasize other relational features than do diagrams. In textual representations information is organized sequentially, preserving temporal and logical relations (cf. Larkin & Simon, 1987), rather than topological and structural relations. This has consequences for the way representations permit accessing and processing of information: diagrams allow simultaneous access (Koedinger & Anderson, 1990; Larkin & Simon, 1987)), whereas in order to access and process a text, the reader has to keep certain elements of the text highly activated in working memory, while comparing newly encountered elements with those held in working memory (Glenberg, Meyer, & Lindem, 1987). Keeping elements activated is thought to burden working memory considerably (Leung et al., 1997; Sweller, van Merriënboer, & Paas, 1998).

$$\boxed{p(\text{PIN}=5526) = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}}$$

Figure 2-3. Equation representing PIN-code problem

Figure 2-3 displays an arithmetical representation of the PIN-code problem. Again, the representation is informationally equivalent with the previous representations, although recognizing the parallels may strongly depend on the student's knowledge of the meaning of arithmetical representations. One needs to know, for example, the *conceptual* meaning of the multiplication sign. In arithmetical representations the underlying principle or concept is not as explicit as in diagrams and texts, and as a result most students tend to view mathematical symbols (e.g., multiplication signs) purely as indicators of which operations to perform on adjacent numbers (Atkinson et al., 2003; Cheng, 1999; Greenes, 1995; Nathan et al., 1992; Niemi, 1996; Ohlsson & Rees, 1991).

2.1 Representational format and learning

How information is organized in an external representation is assumed to influence learning and understanding. Gaining a full understanding of a domain requires students to acquire meaningful schemata. Sweller (1989, p. 458) defined a schema as "...a cognitive construct that permits problem solvers to recognize problems as belonging to a particular category requiring particular moves for solution". A complete schema therefore rests on three pillars: conceptual knowledge, procedural knowledge, and situational knowledge. *Conceptual knowledge* is "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson et al., 2001, p. 346). Conceptual knowledge develops by establishing relationships between pieces of information or between existing knowledge and new information. *Procedural knowledge* is "the ability to execute action sequences to solve problems" (Rittle-Johnson et al., 2001, p. 346). *Situational knowledge* (de Jong & Ferguson-Hessler, 1996) enables students to analyze, identify, and classify a problem, to recognize the concepts that underlie the problem, and to decide which operations need to be performed to solve the problem.

Schemata can be acquired by performing cognitive activities such as selecting, organizing, and integrating information (Mayer, 2003, 2004; Shuell, 1986, 1988; Sternberg, 1984). *Selecting* involves recognizing which information is relevant and which is not. *Organizing* involves combining pieces of information into a coherent and internally connected structure (e.g., a mental representation). *Integrating* refers to relating newly acquired knowledge to already existing knowledge structures (prior knowledge).

The representational format in which instructional material is presented might influence the way students select, organize, and integrate this information. If representational formats have differential effects on these cognitive activities, this might result in different emphases on conceptual, procedural, and situational aspects of the students schemata. For example, diagrams summarize or highlight essential information, make relations among elements explicit, and organize information into coherent structures (Levin, 1981; Levin et al., 1987; Levin & Mayer, 1993). Therefore, the features of diagrams seem particularly suited for the acquisition of conceptual knowledge. In the case of arithmetical representations, the emphasis is on operational aspects rather than on conceptual or situational aspects. This is assumed to result in schemata that rely more on procedural knowledge. With respect to textual representations the use of natural language facilitates of the relation of information in the text to everyday experiences and situations. Textual representations allow students to analyze and understand problem statements, in particular stressing situational domain aspects.

Combining two or more representational formats into what is called a multiple representation (e.g., van Someren et al., 1998) is assumed to have some additional effects on schema construction processes (Ainsworth, 1999, 2006; Seufert, 2003). First, different formats can complement each other; for example, combining an equation and a diagram might be helpful in focusing the students' attention on not only operational aspects but also conceptual aspects of the domain. Second, one representation might constrain the interpretation of the other. For example, when an arithmetical representation such as an equation is accompanied by a textual representation, the latter might help students to better understand the equation. Third, students' integration of information from different representations is thought to support the construction of deeper understanding (Ainsworth, 1999, 2006; van der Meij & de Jong, 2006). However, combining different formats of equivalent information is not always beneficial for learning. It may interfere with cognitive processing (e.g., split-attention effects), and a multiple representation may contain redundant information, which is assumed to increase cognitive load (e.g., Leung et al., 1997).

2.2 Representational format and cognitive load

Cognitive load theory (CLT) is based on the idea that working memory capacity is limited (Miller, 1956). CLT distinguishes between three types of working memory load: intrinsic, extraneous, and germane load. *Intrinsic load* is generated by the complexity (element interactivity) of the learning material. *Extraneous load* is determined by the way in which the material is organized and presented (e.g., diagrams or text formats). *Germane load* refers to load caused by mental activities relevant to schema acquisition, such as organizing the material and relating it to prior knowledge (DeLeeuw & Mayer, 2008; Paas, Renkl, & Sweller, 2004; Sweller et al., 1998). Cognitive load principles may support or even determine decisions as to which representational format to use (Leung et al., 1997). Carlson, Chandler, and Sweller (2003) found that where intrinsic load was low (low element interactivity), the extraneous load caused by instructional format was of little consequence. However, in the case of high intrinsic load (high element interactivity), the extraneous load caused by instructional format turned out to be critical. Here, students who were presented with diagrams performed better and reported significantly lower levels of cognitive load compared to students presented with textual representations. Yet, empirical findings about the relation between representational format, cognitive load, and learning outcomes are far from straightforward: the advantage of a particular format in learning is not universal, but depends on a complex interaction among the nature of the task and the material, the student's ability, prior knowledge, and practice time. For example, Dee-Lucas and Larkin (1991) found that learning with verbal material was superior to learning with equivalent equations. Leung et al. (1997) replicated these findings but also found that this effect only applies to less able students. Moreover, as material required more complex verbal statements but simpler equations, equations turned out to be more effective, suggesting lower levels of cognitive load. Finally, Leung et al. also found that additional practice with equations reduced cognitive load and increased performance.

2.3 Assessing the effects of representational format

From the previous sections it should be clear that decisions as to which representational format to use in instruction are particularly critical in the case of students with little or no prior knowledge who are dealing with complex instructional material (high intrinsic load). The aim of the current study was to investigate the effects of different representational formats on learning combinatorics and probability theory. The PIN-code example presented earlier is a typical problem for this domain. Understanding and solving problems like these requires students to process many interrelated elements concurrently (high interactivity and therefore high intrinsic load). It has been demonstrated that instructional material in this domain can be represented in several informationally equivalent ways. In this study, five conditions were compared: three conditions each using a single external representational format (Diagram, Arithmetic, or Text), and two conditions using multiple external representations, that is combinations of single representational formats (Text + Arithmetic or Diagram + Arithmetic). In theory, more combinations of external representations would be possible (e.g., diagram plus text), but due to screen size limitations, these combinations were not feasible without severely hampering the readability of information presented to subjects in the study. The effects of format could be isolated by varying only the representational format and by keeping the different representations informationally equivalent. The main focus was on the effects of formats on schema construction, cognitive load, and interactiveness (students manipulating interactive representations).

3 METHOD

3.1 Participants

A total of 123 students participated in the study: 61 boys and 62 girls. The average age of the participants was 15.61 years (*SD*=0.59). Three participants were excluded from the analyses because their post-test scores deviated more than 2 SDs from the mean scores within their condition. The participants participated in the experiment during regular school time, so that participation was obligatory.

3.2 Design

The experiment employed a between-subjects pre-test post-test design, with the representational format in which the domain was presented (diagram, arithmetic, text, a combination of text and arithmetic, or a combination of diagram and arithmetic) as the independent variable. The distribution of the participants across conditions is displayed in Table 2-1.

Table 2-1
Number of participants per condition

	Representational format				
	Diagram	Arithm	Text	Text+ Arithm	Diagram+ Arithm.
Participants	24	25	24	24	23

3.3 Domain

The domain of instruction was combinatorics and probability theory. Combinatorics can be used to determine the number of combinations that can be made with a certain set or subset of elements. Probability theory can be used to calculate the chance that a certain combination will be observed empirically. The PIN-code problem presented in the introductory section is a typical problem for this domain. In order to determine the number of possible combinations, one also needs to know 1) whether elements may occur repeatedly in a combination (replacement) and 2) whether the order of elements in a combination is of interest (order). On the basis of these two criteria, four problem categories can be distinguished (for an overview, see Figure 2-4).

		ORDER IMPORTANT?	
		Yes	**No**
REPLACEMENT?	**No**	Category 1: No replacement; Order important	Category 2: No replacement; Order not important
	Yes	Category 3: Replacement; Order important	Category 4: Replacement; Order not important

Figure 2-4. Problem categories within the domain of combinatorics

The PIN-code example matches category 3 (replacement; order important).

3.4 Learning environment

The instructional approach used in this study is based on inquiry learning (de Jong, 2005, 2006). Computer-based simulation is a technology that is particularly suited for inquiry learning. Computer-based simulations contain a model of a system or a process. The student is enabled to induce the concepts and principles underlying the model by manipulating the input variables and observing the resulting changes in output values (de Jong & van Joolingen, 1998).

The learning environments used in the current study were created with SimQuest authoring software (van Joolingen & de Jong, 2003). A learning environment was developed for each experimental condition. The learning environments contained simulations in which students could manipulate the number of possible options (generally indicated by N) and the number of element selections (usually indicated by K). The output values displayed the corresponding situation and probability of an event, given the input variables. All five learning environments were identical to each other, except for the representational format of the simulation output. The representational format was diagram, arithmetic, text, a combination of text and arithmetic, or a combination of diagram and arithmetic. The learning environments consisted of five sections. Four of these sections were devoted to each of the four problem categories within the domain of combinatorics. The fifth section aimed at integrating these four problem categories. Each section used a different cover story, that is, an everyday life example of a situation in which combinatorics and probability played a role. Each cover story exemplified the problem category treated in that section. In the fifth (integration) section, the cover story applied to all problem categories.

Each of the five sections contained a series of task assignments (both open-ended and multiple-choice), all based on the cover story for that particular section. Students could use the simulations to complete the assignments. The assignments involved determining which problem category matched the given cover story (situational knowledge), calculating the probability in a given situation (procedural knowledge), and selecting the description that matched the relation between variables most accurately (conceptual knowledge). In the case of the multiple-choice items, the students received feedback from the system about the correctness of their answer. If the answer was wrong, the system offered hints about what was wrong with the answer. Students then had the opportunity to select another answer. For the open-ended questions, students received the correct answer after completing and closing the question.

In this study we measured cognitive load. The literature mentions several ways to measure cognitive load (Ayres, 2006; Brünken, Plass, & Leutner, 2003, 2004; DeLeeuw & Mayer, 2008; Paas, Tuovinen, Tabbers, & Van Gerven, 2003). The current study employed self-reports; after each section a questionnaire regarding perceived cognitive load appeared on the screen. Self-reports have been found to be valid, reliable, unintrusive, and sensitive to relatively small differences in cognitive load (e.g., Ayres, 2006; Paas et al., 2003). The questionnaire used in the current study constisted of six items (see Table 2-2). One item intended to measure overall load. This item was adopted from a study by Paas (1992). The set of remaining items was an adapted and extended version of the SOS-scale (Swaak & de Jong, 2001). These five items were intended to be indicative of intrinsic (1 item), extraneous (3 items), and germane load (1 item). The students indicated their amount of mental effort on 9-point Likert scales. Each time the cognitive load questions were presented, they appeared in a different order.

Table 2-2
Cognitive load items

Type of cognitive load	Item
Intrinsic load (IL)	How easy or difficult do you consider probability theory at this moment?
Extraneous load 1 (EL1)	How easy or difficult is it for you to work with the learning environment?
Extraneous load 2 (EL2)	How easy or difficult is it for you to distinguish important and unimportant information in the learning environment?
Extraneous load 3 (EL3)	How easy or difficult is it for you to collect all the information that you need in the learning environment?
Germane load (GL)	How easy or difficult was it to understand the simulation?
Overall load (OL)	Indicate on the scale the amount of effort you had to invest to follow the last simulation.

3.5 Knowledge measures

Two knowledge tests were used in this experiment: a pre-test and a post-test. These tests contained 12 and 44 items respectively. An overview of the test items and underlying knowledge types is presented in Table 2-3.

Table 2-3
Overview of test items

Knowledge type	Description	Number and type of items
Pretest		
Conceptual	Items about relations between variables	4 mc[a]
Procedural	Solving combinatorial problems	8 open
Total		12 items
Post-test		
Conceptual	Items about relations between variables	4 open and 21 mc[a]
Procedural	Solving combinatorial problems	14 open
Situational	Analyzing, identifying, and classifying problems	1 open and 4 mc[a]
Total		44 items

[a] mc = multiple choice

In cases where students had to calculate the outcome of an item, a calculator was provided on-screen. Both tests were administered via the internet. The questions were presented screen-by-screen without allowing participants to skip questions or turn back to previous questions. Student responses on both multiple-choice and open-ended items were collected and recorded electronically.

3.6 Procedure

The experiment was carried out in a real school setting in one three hour session including a 15 minute break. Students worked individually. They were told that they could work at their own pace, and that the three hours would be more than enough to complete all assignments. The participants started the session by logging on to the electronic pre-test. It was announced that the post-test would contain more items of greater difficulty than the pre-test, but that the pre-test items nonetheless would give an indication of what kind of items to expect on the post-test. After completing the pre-test the participants received a printed introductory text introducing the domain. Along with the introductory text the participants received information about how to enter the learning environment. After finishing the last section of the learning environment, the participants received log-on instructions for the post-test environment.

4 RESULTS

4.1 Pre-Test

The overall pre-test scores are summarized in Table 2-4.

Table 2-4
Pre-test scores

	Representational format									
	Diagram		Arithmetic		Text		Text + Arithmetic		Diagram + Arithmetic	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Pre-test	7.67	1.90	8.20	1.80	8.29	2.03	8.38	1.58	8.00	1.24

A one-way ANOVA performed on the pre-test scores established that there were no differences between conditions, $F(4,119) = 0.63$, *ns*.

4.2 Post-Test

The post-test results are displayed in Table 2-5. All post-test measures were analyzed by one-way ANOVAs with representational format as factor.

Table 2-5
Post-test scores

	Representational format									
Knw ldge	Diagram		Arithmetic		Text		Text + Arithmetic		Diagram + Arithmetic	
type	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Conc	17.17	3.10	18.04	2.91	18.17	2.75	19.08	2.21	18.04	2.21
Int	9.12	2.01	10.00	1.94	9.96	2.07	10.58	1.50	9.83	1.64
Proc.	5.83	2.73	6.80	3.06	6.75	2.45	8.08	2.84	6.13	2.12
Situ.	3.54	0.93	3.40	1.12	3.38	1.10	3.75	0.85	3.70	1.02
TOT	26.54	4.91	28.24	5.75	28.29	4.97	30.92	4.32	27.87	4.07

Analysis of *post-test total scores* showed a significant effect of representational format, $F(4,119) = 2.57$, $p < .05$. A post-hoc least significant difference (LSD) analysis revealed that participants in the Text + Arithmetic condition outperformed participants in the Diagram condition ($p < .01$), and participants in the Diagram + Arithmetic condition ($p < .05$).

With regard to *conceptual knowledge* no main effect of format, $F(4,119) = 1.56$, *ns*, was found. Neither was a main effect of format on intuitive knowledge observed, $F(4,119) = 1.91$, *ns*, was found

Analysis of *procedural knowledge* revealed a significant main effect of representational format, $F(4,119) = 2.52$, $p < .05$. A post-hoc LSD analysis showed that participants in the Text + Arithmetic condition outperformed participants in the Diagram condition ($p < .01$), and participants in the Diagram + Arithmetic condition ($p < .05$).

With regard to *situational knowledge* items no main effect of representational format, $F(4,119) = 0.66$, *ns*, was found.

4.3 Cognitive load

In the learning environment, the participants had to indicate on 9-point Likert scales the amount of perceived difficulty (cognitive load) of: the domain in general (intrinsic load (IL)); working with the learning evironment (extraneous load 1 (EL1)); distinguishing between important and unimportant information (extraneous load 2 (EL2)); collecting needed information (extraneous load 3 (EL3)); understanding the simulation (germane load (GL)); and the amount of invested mental effort (overall load (OL)). The results are displayed in Table 2-6.

Table 2-6
Cognitive load measures

	Representational format									
	Diagram		Arithmetic		Text		Text + Arithmetic		Diagram + Arithmetic	
Type	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
IL	4.37	1.55	3.65	1.51	3.96	1.31	3.11	1.14	3.52	1.14
EL1	4.34	1.49	3.40	1.14	3.68	0.90	3.38	1.22	3.45	0.95
EL2	4.36	1.62	3.14	1.21	3.68	0.95	3.35	1.51	3.22	1.05
EL3	4.28	1.45	3.54	1.26	3.62	1.00	3.29	1.03	3.51	1.23
GL	4.40	1.67	3.51	1.25	3.75	1.05	3.52	1.30	3.61	1.06
OL	4.36	1.56	3.26	1.23	3.69	1.38	3.29	1.27	3.43	1.03

All cognitive load measures were analyzed by one-way ANOVAs with condition as factor.

Regarding the question of how easy or difficult the participant considered probability theory (*intrinsic load*), a main effect of representational format was found, $F(4,118) = 2.70$, $p < .05$. Post-hoc LSD analyses showed that participants in the Diagram condition considered probability theory more difficult than participants in the Text + Arithmetic condition ($p < .01$) and participants in the Diagram + Arithmetic condition ($p < .05$). Furthermore, participants in the Textual condition considered probability theory more difficult than participants in the Text + Arithmetic condition ($p < .05$).

Differences between conditions with regard to extraneous load were observed as well, $F(4,118) = 2.89$, $p < .05$. Participants in the Diagram condition experienced higher levels of extraneous load compared to participants in the Arithmetic condition ($p < .01$), the Text + Arithmetic condition ($p < .01$), and the Diagram + Arithmetic condition ($p < .01$). Analysis of the extraneous load sub measures (EL 1, EL 2, and EL 3) showed that the differences between conditions with regard to extraneous load, in general, could be entirely attributed to EL 1, ($F(4,118) = 3.55$, $p < .01$), that is: participants in the Diagram condition perceived their learning environment as more difficult compared to participants in the Arithmetic condition ($p < .01$), the Text + Arithmetic condition ($p < .01$), and the Diagram + Arithmetic condition ($p < .01$).

With regard to understanding the simulations (*germane load*), differences were found between conditions, $F(4,118) = 2.89$, $p < .05$. Participants in the Diagram condition reported having more difficulty understanding their simulations as compared to participants in the Arithmetic condition ($p < .01$), the Text + Arithmetic condition ($p < .01$), and the Diagram + Arithmetic condition ($p < .05$).

Furthermore, a difference between conditions was found concerning the amount of effort participants had to invest to complete the learning task (*overall *load*), $F(4,117) = 2.85$, $p < .05$. Participants in the Diagram condition had to invest more effort than participants in the Arithmetic condition ($p < .05$), and the Text + Arithmetic condition ($p < .01$).

4.4 Interactiveness

A summary of the time spent in the learning environment (minutes), and the total number of manipulations performed in the simulations is displayed in Table 2-7. Three outliers have been excluded from the analysis of the number of manipulations,. Furthermore, due to technical reasons the manipulation data of 25 students were not available. This loss of data was equally distributed over experimental conditions. The data for the remaining 95 participants are displayed in Table 2-7.

Table 2-7
Time-on-task (minutes) and number of manipulations performed in the simulations

	Representational format									
	Diagram		Arithmetic		Text		Text + Arithmetic		Diagram + Arithmetic	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Time	30.0	8.6	32.5	7.6	34.2	11.1	36.6	8.8	33.0	9.5
Manip.	12.5	9.2	23.6	15.3	17.0	12.3	21.6	11.1	17.9	7.9

A one-way ANOVA showed that there were no differences between conditions with regard to time-on-task, $F(4,119) = 1.62$, *ns*. With regard to the number of manipulations performed in the simulations, a significant difference between conditions was found, $F(4,90) = 2.74$, $p < .05$. Post-hoc LSD analyses showed that participants in the Diagram condition performed fewer manipulations than participants in the Arithmetic condition ($p < .01$) and participants in the Text + Arithmetic condition ($p < .05$).

5 DISCUSSION AND CONCLUSION

Decisions on which representational format to use in instruction are particularly critical in the case of students with little or no prior knowledge who must deal with complex instructional material. The aim of the current study was to investigate the effects of different representational formats (Diagram, Arithmetic, Text, or the combinations, Text + Arithmetic or Diagram + Arithmetic) on learning combinatorics and probability theory. The main focus was on the effects of formats on schema construction, cognitive load, and interactiveness. It was expected that different formats would have different effects on the nature of the constructed schemata. Diagrams were expected to result in schemata emphasizing conceptual knowledge. Arithmetical representations were expected to lead to an emphasis on procedural knowledge, whereas textual representations were assumed to stress the situational aspects of the domain. Multiple representations were thought to serve complementing and constraining functions. In general, research findings have suggested that diagrams help to reduce extraneous cognitive load in complex domains.

The idea of diagrams being most beneficial for learning has found its way into everyday practice in classrooms. Tree diagrams are considered a powerful tool for teaching combinatorics and probability theory (e.g., Fischbein, 1987; Greer, 2001).

However, the major finding of this study is that the use of tree diagrams in this domain is not as beneficial as generally thought. Although the students had prior experience with tree diagrams, the results show that learning with tree diagrams leads to poorer performance at solving problems mathematically, and to higher levels of cognitive load. The measures intended to be indicative of intrinsic, extraneous, and germane load showed that participants presented with tree diagrams considered the domain more difficult (intrinsic load), found the learning environment more difficult (extraneous load), had more difficulty understanding the simulations (germane load), and had to invest more mental effort to complete their learning task. It was also found that the addition of equations to the tree diagrams (Diagram + Arithmetic condition) led to a significant reduction of intrinsic, extraneous, and germane cognitive load, but not to better learning outcomes.

The best learning outcomes were obtained by students who were presented with a combination of text and equations (Text + Arithmetic condition). Their outcomes were significantly better than those of students presented with tree diagrams or with a combination of tree diagrams and equations. Students in all experimental conditions demonstrated equal levels of understanding of domain concepts, variables, and their mutual relations (concepual knowledge) and were equally able to analyze, identify, and classify problems, to recognize the concepts underlying these problems, and to decide which operations needed to be performed to solve the problems (situational knowledge). However, students who had been presented with a combination of text and equations during the learning phase turned out to be better at finding mathematically correct solutions to these problems (procedural knowledge).

The results indicate that the differential effects of formats on the nature of schema construction are not as straightforward as expected (e.g, diagrams resulting in schemata emphasizing conceptual knowledge, arithmetical representations stressing procedural knowledge, and so on). The effects of combining different representational formats into multiple representations are not straightforward either. The observation that students presented with a combination of text and equations tended to outperform students presented with text only or equations only suggests a postive effect of the combination of these formats. However, the data do not allow interpretation in terms of complementing, constraining, and arriving at deeper understanding by integrating. In the condition where equations were added to tree diagrams, the learning outcomes remained unaffected, but a significant reduction of cognitive load was observed. It remains unclear whether this reduction of mental load was caused by complementing and constraining functions. It is possible that students in this multiple representation condition ignored the tree diagrams and focused on the equations.

Rogers (1999) suggested that interactive representations can reduce the amount of “low-level” cognitive activities (e.g., drawing and redrawing) normally required when learning, and in turn allow students to focus their cognitive resources on more “high-level” cognitive tasks, such as exploring more of the problem space. It was found that students presented with tree diagrams did not perform as many manipulations on the representations as did students in the other conditions. This might be an indication that tree diagrams inhibit exploration of the problem space.

In short, the major finding of this study is that the use of tree diagrams in this domain is not as beneficial as generally thought: the learning outcomes are relatively poor, the experienced levels of cognitive load are high, and the format seems to inhibit problem space explorations. The question remains of accounting for these findings. A possible explanation is provided by Tabachneck-Schijf, Leonardo, and Simon (1997). They argue that experts in particular benefit from diagrams because for them diagrams serve as an aid to access information stored in long term memory. Moreover, diagrams help to decrease the expert's working memory load, because elements of information that are displayed in the diagram do not have to be kept activated in working memory all the time. This frees up cognitive resources the expert can devote to reasoning and problem solving instead. This implies that diagrams are more suited for people who already possess the relevant schemata (e.g., math teachers), and just need the diagrams as mnemonic devices. This explanation also suggests that tree diagrams are less suited for students to infer reasoning steps from, because the reasoning steps depicted in tree diagrams are quite implicit and require advanced knowledge about the diagram in order to identify them. This might be a disavantage in domains like combinatorics and probability theory, where problem solving requires a set of reasoning steps that are taken in a specific order. This would explain the advantage of the Text + Arithmetic format. In the textual part of the representation the students are taken by the hand as it were, and led step-by-step through an explicit and sequential line of reasoning described in everyday language, followed by an equation concisely repeating these steps in an arithmetical (and also sequential) way. The Text + Arithmetic format preserves the strictly sequential nature of problem solving in this domain, whereas the often-claimed advantage of diagrams, allowing for simulataneous processing, turns into a disadvantage.

Chapter 3

Influence of student-generated representations on learning[1]

Abstract

The aim of the current study was to examine the effects of providing support in the form of tools for constructing representations, and in particular the differential effects of the representational format of these tools (conceptual, arithmetical, or textual) in terms of perceived affordances and learning outcomes. The domain involved was combinatorics and probability theory. A between-subjects pretest-posttest design was applied with secondary education students randomly distributed over four conditions. Participants completed the same tasks in a simulation-based learning environment. Participants in three experimental conditions were provided with a representational tool that could be used to construct a domain representation. The experimental manipulation concerned the format of the tool (conceptual, arithmetical, or textual). Participants in a control condition did not have access to a representational tool. Data from 127 students were analyzed. It was found that the construction of a domain representation significantly improved learning outcomes. The format in which students constructed a representation did affect neither the learning outcomes nor the quality of the created domain representations. The arithmetical format, however, was the least stimulating for students to engage in externalizing their knowledge.

[1] This chapter is adapted from Kolloffel, B., Eysink, T. H. S., & de Jong, T. (submitted). *The influence of student-generated domain representations on learning combinatorics and probability theory*

1 INTRODUCTION

Learning to understand science and mathematics is hard for many students. The current study seeks to facilitate the learning process by offering students tools to create external representations while learning. It does so in a subdomain of mathematics which is known to be notoriously difficult: combinatorics and probability theory.

One of the more general reasons for students' difficulties with science and mathematics problems is that novices often have a tendency to focus on superficial details rather than on understanding the principles and rules underlying a science or mathematics domain (Chi et al., 1981; de Jong & Ferguson-Hessler, 1986; Reiser, 2004). Science and mathematics problems require students to go beyond the superficial details in order to recognize the concepts and structures that underlie the problem and to decide which operations are required to solve it (e.g., Fuchs et al., 2004). In the case of probability instruction, for example, identifying the approach that needs to be taken to solve a problem depends a great deal on correct classification of the problem (Lipson et al., 2003).

A second reason for students' difficulties is that the abstract and formal nature of often used arithmetical representations does not illustrate the underlying principles or concepts as explicitly as pictorial and textual representations. Most students tend to view mathematical symbols (e.g., multiplication signs) purely as indicators of which operations need to be performed on adjacent numbers, rather than as reflections of principles and concepts underlying these procedures (Atkinson et al., 2003; Cheng, 1999; Greenes, 1995; Nathan et al., 1992; Niemi, 1996; Ohlsson & Rees, 1991). Therefore, they easily lose sight of the meaning of their actions. Correctly processing formal notations thus becomes an end in itself (Cheng, 1999), not for the purpose of understanding and communicating concepts but for getting high scores on tests (Greeno & Hall, 1997). Learning of arithmetical procedures without conceptual understanding tends to be error prone, easily forgotten, and not readily transferable (Ohlsson & Rees, 1991).

Third, the formal, abstract way in which subject matter is represented makes it hard for students to relate the subject matter to everyday life experiences. Fuson, Kalchman, and Bransford (2005) argue that the knowledge students bring into the classroom is often set aside in mathematics instruction and replaced by procedures that disconnect problem solving from meaning making. The integration of theory and everyday life experience is particularly important in probability and combinatorics, because the principles of probability often appear to conflict with students' experiences and how they view the world (Garfield & Ahlgren, 1988; Kapadia, 1985). The conflicts arise because probabilities do not always match students' conceptions and intuitions (e.g., Batanero & Sanchez, 2005; Fischbein, 1975; Greer, 2001). An example of a misconception is the gambler's fallacy, that is, the belief that the outcome of a random event can be affected by (and therefore predicted from) the outcomes of previous events.

These reasons are by no means exhaustive, but summarize some of the main problems encountered in the instruction of combinatorics and probability theory. What can be learned from these points to help improve instruction in this domain? One of the suggestions that follows from this list is that the (abstract and formal) way in which information is presented plays a critical role. The effects of format were tested in a previous study, where different formats (tree diagrams, mathematical equations, texts, or combinations of these) were compared in terms of their effects on learning outcomes and cognitive load (Kolloffel, Eysink, de Jong, & Wilhelm, in press). Learning outcomes improved when using a text describing solution steps on the basis of everyday life situations and simultaneously presenting an equation repeating the same information in a mathematical format.

The aim of the current study is to find out whether more can be done to support and scaffold students to help them overcome the problems described above. A promising approach traditionally found to help students gain better understanding and focus more on the underlying principles and concepts of the domain is for students themselves to construct a representation of the domain, as by writing a summary, creating a drawing, building a runnable computer model, or constructing a concept map.

1.1 Constructing representations

Constructing representations can have different purposes. For example, for students with advanced levels of domain knowledge, constructing a representation may serve as an aid to accessing information stored in long term memory and as a summary of their processing, which decreases working memory load and thus helps them to concentrate on reasoning (Tabachneck-Schijf et al., 1997). For students unfamiliar with the domain, constructing representations can support learning and understanding (Greeno & Hall, 1997; Lesh & Lamon, 1992). Gaining a full understanding of a domain requires students to recognize which information is relevant, to combine pieces of information into a coherent and internally connected structure (e.g., a mental representation), and to relate newly acquired knowledge to prior knowledge (Mayer, 2003, 2004; Shuell, 1986, 1988; Sternberg, 1984). Cox (1999) argues that the process of constructing a representation elicits self-explanation effects and consists of dynamic iterations and interactions between the constructed representations and mental representations and therefore helps students to refine and disambiguate their domain knowledge.

Evidence from studies in which students (collaboratively) constructed representations indicates that the format in which students construct representations plays a significant role in knowledge construction processes (e.g., Gijlers & de Jong, submitted; Suthers & Hundhausen, 2003; van Drie et al., 2005). Representational formats can differ with regard to the affordances students perceive.

In the first place, the properties of representations influence which information is attended to and how people tend to organize, interpret, and remember the information (Ainsworth & Loizou, 2003; Cheng, 1999; Larkin & Simon, 1987; Zhang, 1997). This is called *constraining* (Ainsworth, 2006; Scaife & Rogers, 1996; Stenning & Oberlander, 1995), *representational bias* (Utgoff, 1986), or *representational guidance* (Suthers, 2003; Suthers & Hundhausen, 2003). For example, constructing concept maps directs attention to concepts and their mutual relationships (Nesbit & Adesope, 2006) and using formal arithmetical representations may focus attention on procedures rather than on principles and concepts (Atkinson et al., 2003; Cheng, 1999; Ohlsson & Rees, 1991).

In the second place, the perceived affordances of formats for expressing knowledge also depend on familiarity with the format and domain; some formats may seem easier or more appropriate for constructing a representation than others. For example, to students with advanced levels of mathematical knowledge, symbols and formulas may be the easiest and most appropriate way to express their knowledge. To them, these formats are a common and efficient way to express both procedures and underlying principles and concepts. For students relatively new to the domain, using symbols and formulas to construct a representation might seem too difficult or inappropriate, resulting in incorrect and/or incomplete representations, or even a failure to construct a representation at all. Such students may lack the knowledge to use this formal language or may be prone to the misconception that those formats reflect only procedures and not underlying concepts or principles. Tarr and Lannin (2005) found that in conditional probability instruction, students initially avoid using conventional ways of representing probabilities (i.e., using ratios or odds, or formal numerical probabilities). Instead they use alternative forms of representation, such as textual statements. When they reach more advanced levels of knowledge, some of them then start using more conventional representations.

1.2 Research questions

The aim of the current study was to examine the effects of providing support in the form of tools for constructing representations, and in particular the differential effects of the representational format of these tools in terms of perceived affordances and learning outcomes. The following questions guided this study. Do representational formats have differential effects on the likelihood that students use the support and engage in constructing representations? Does format have differential effects on the quality of the representations students construct? Does the construction of a representation of a domain lead to better learning outcomes than not constructing a representation? And, if students construct a representation, does format have differential effects on domain understanding?

On the basis of the arguments outlined in the previous section, it is hypothesized that constructing a representation of a domain is beneficial for learning and understanding (e.g., Cox, 1999). The format in which such a representation is constructed is assumed to have differential effects on knowledge construction and domain understanding. Three formats for constructing representations were compared: (a) conceptual (i.e., concept map), (b) arithmetical, and (c) textual.

Constructing a conceptual representation like a concept map is thought to focus the student's attention on the identification of concepts and their mutual relationships (Nesbit & Adesope, 2006), which is hypothesized to result in enhanced levels of knowledge about the conceptual aspects of the domain. A concept map is not a very complicated format, in particular when the number of concepts and relations is not too large (van Drie et al., 2005).

Constructing representations in an *arithmetical* format is assumed to draw the student's attention primarily to operational aspects. Regarding the likelihood that students will construct a representation using the arithmetical format, it is hypothesized that compared to other formats students may have difficulty constructing arithmetical representations (Tarr & Lannin, 2005).

Constructing *textual* representations is assumed to direct the student's attention to conceptual and situational aspects, although the conceptual issues might not be as strongly stressed as for students who construct a conceptual representation. It is expected that students will not experience too many difficulties using this format. Overall, in educational settings this is the most commonly used format. In addition, this domain allows for representation in everyday language, not least because combinatorial problems can easily be described in terms of everyday life situations.

2 METHOD

2.1 Participants

In total, 133 secondary education students, 65 boys and 62 girls (six students did not indicate their gender), participated. The average age of the students was 14.63 years (SD =.62). The domain of combinatorics and probability theory is part of the regular curriculum and the experiment took place a few weeks before this subject would be covered. The students completed the experiment during regular school time; therefore, participation was obligatory. They received a grade based on their post-test performance.

2.2 Design

The experiment employed a between-subjects design with four conditions: three experimental conditions and one control condition. All students (including those in the control condition) had to complete the same tasks: completing a pre-test, working through a simulation-based learning environment, and completing a post-test. The only difference between the control condition and the experimental conditions was that students in the experimental conditions were asked to construct a representation of the domain. Their learning environments were equipped with an additional tool: the representational tool. The difference between the three experimental conditions concerned the format of the representational tool, which could be conceptual, arithmetical, or textual. The students in the experimental conditions were informed in advance about the (general) beneficial effects on learning of constructing representations. Students were assigned randomly to conditions. Afterwards, six students were excluded from the analyses because they missed one or more experimental sessions. Of the remaining 127 students, 33 were in the Conceptual condition, 30 in the Arithmetical condition, 32 in the Textual condition, and 32 in the Control condition.

2.3 Domain

The domain of instruction was combinatorics and probability theory. An example of a problem in this domain is: what is the probability that a thief will guess the 4-digit PIN-code of your credit card correctly the first try? The essence of combinatorics is determining how many different combinations can be made with a certain set or subset of elements. In order to determine the number of possible combinations, one also needs to know 1) whether elements may occur repeatedly in a combination (replacement) and 2) whether the order of elements in a combination is of interest (order). On basis of these two criteria, four so-called problem categories can be distinguished. The PIN-code example matches the category “replacement; order important”. When the number of possible combinations is known, the probability that one or more combinations will occur in a random experiment can be determined.

2.4 Learning environment

The instructional approach used in this study is based on inquiry learning (de Jong, 2005, 2006). Computer-based simulation is a technology that is particularly suited for inquiry learning. Computer-based simulations contain a model of a system or a process. By manipulating the input variables and observing the resulting changes in output values the student is enabled to induce the concepts and principles underlying the model (de Jong & van Joolingen, 1998).

The learning environment used in the current study, called Probe-XMT (see Figure 3-1), was created with SIMQUEST authoring software (van Joolingen & de Jong, 2003).

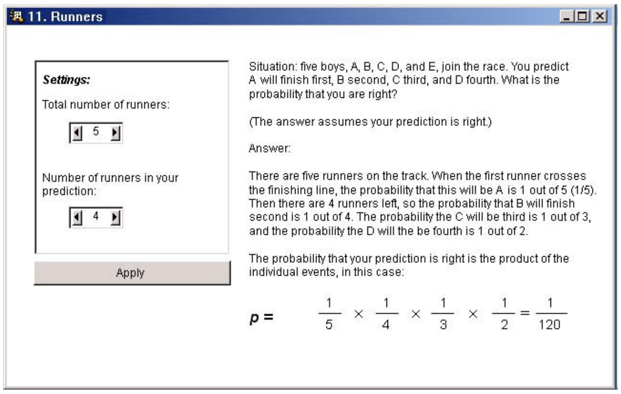


Figure 3-1. Screen dump Probe-XMT simulation

In the box on the left-hand side of the simulation (see Figure 3-1), students could manipulate input variables. On the right-hand side of the simulation the resulting effects of the manipulations on the output values could be observed. In this case the output consisted of a text and an equation that changed whenever the input variables were changed. Probe-XMT consisted of five sections. Four of these sections were devoted to each of the four problem categories within the domain of combinatorics. The fifth section aimed at integrating these four problem categories. Each section used a different cover story, that is, an everyday life example of a situation in which combinatorics and probability played a role. Each cover story exemplified the problem category treated in that section. In the fifth (integration) section, the cover story applied to all problem categories.

Each of the five sections in the learning environment contained a series of questions (both open-ended and multiple-choice items), all based on the cover story for that particular section. These questions involved determining which problem category matched the given cover story (situational knowledge), calculating the probability in a given situation (procedural knowledge), and selecting a description that matched the relation between variables most accurately (conceptual knowledge). In the case of the multiple-choice items, the students received feedback from the system about the correctness of their answer. If the answer was wrong, the system offered hints about what was wrong with the answer. Students then had the opportunity to select another answer. In the case of the open-ended questions, students received the correct answer after completing and closing the question.

Most of the questions were accompanied by simulations that could be used to explore the relations between variables within the problem category. In the simulations, students could manipulate variables and observe the effects of their manipulations on other variables. The simulations used a combination of textual and arithmetical representations. This combination of representations was found to have benefits in terms of learning outcomes and mental effort (Kolloffel et al., in press).

The learning environment automatically registered student actions. User actions that were logged included measures such as user path through the learning environment (which parts of the learning environment were opened, when, for how long, and in what sequence) and the number and nature of manipulations carried out in the simulations (how many experiments were carried out and the input values of each experiment).

2.5 Representational tools

Students in the experimental conditions were encouraged to construct a representation of the domain that would be meaningful to themselves and a fictitious fellow student. This representation could be used to summarize principles underlying the domain, the variables playing a role in the domain, and their mutual relationships. Students could create their representations by means of an electronic on-screen representational tool. There were three types of representational tools, one for each experimental condition: (a) a conceptual representational tool, (b) an arithmetical representational tool, and (c) a textual representational tool.

The *conceptual representational tool* (see Figure 3-2) could be used to create a conceptual representation of the domain. Students could draw circles representing domain concepts and variables. Keywords could be entered in the circles. The circles could be connected to each other by arrows indicating relations between concepts and variables. The nature of these relations could be specified by attaching labels to the arrows.

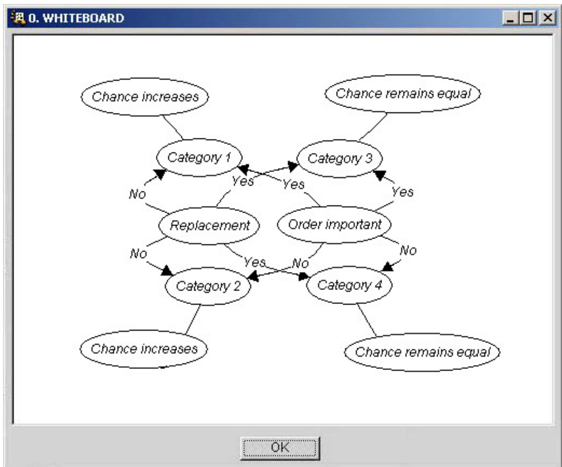


Figure 3-2. Conceptual representational tool

In the *arithmetical representational tool* (see Figure 3-3), students could use variable names (N, K, and P), numerical data, and mathematical operators (division signs, equals signs, multiplication signs, and so on) in order to express their knowledge.

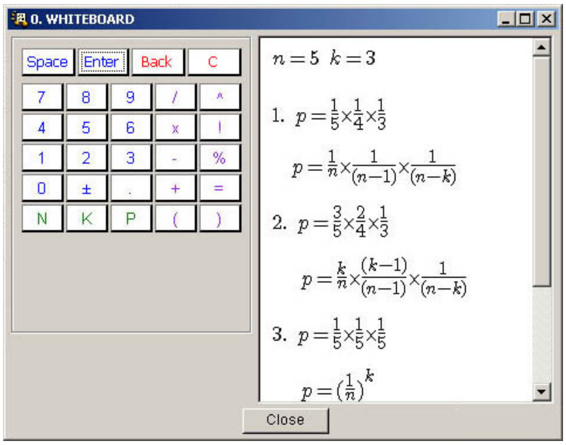


Figure 3-3. Arithmetical representational tool

Finally, the *textual representational tool* (see Figure 3-4) resembled simple word processing software, allowing textual and numerical input.

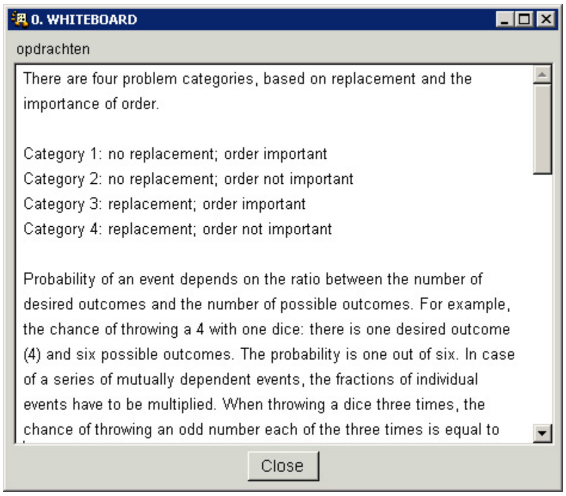


Figure 3-4. Textual representational tool

The contents of the representational tools were stored automatically.

2.6 Knowledge measures

Two knowledge tests were used in this experiment: a pre-test and a post-test. The tests contained 12 and 26 items respectively. The pre-test aimed at measuring the prior knowledge of the students. The post-test aimed at measuring the completeness of students' schemas related to this domain. Sweller (1989, p. 458) defined a mathematical schema as "a cognitive construct that permits problem solvers to recognize problems as belonging to a particular category requiring particular moves for solution". A complete schema therefore rests on three pillars: situational knowledge, conceptual knowledge, and procedural knowledge. Situational knowledge (de Jong & Ferguson-Hessler, 1996) enables students to analyze, identify, and classify a problem, to recognize the underlying concepts, and to decide which operations are required to solve the problem. There were four multiple-choice items measuring this type of knowledge on the post-test (see Figure 3-5 for an example).

You throw a die 3 times and you predict that you will throw two sixes and a one in random order. What is the characterization of this problem? a. order important; replacement b. order important; no replacement c. order not important; replacement d. order not important; no replacement

Figure 3-5. Post-test item measuring situational knowledge

Conceptual knowledge is "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson et al., 2001, p. 346). Conceptual knowledge develops by establishing relationships between pieces of information or between existing knowledge and new information. The post-test contained 13 multiple choice items aiming at measuring conceptual knowledge (see Figure 3-6 for an example).

You play a game in which you have to throw a die twice. You win when you throw a 3 and a 4. Does it matter if these two numbers must be thrown in this specific order? a. Yes, if you have to throw the numbers in a specific order your chance is greater than when the order doesn't matter b. Yes, if you have to throw the numbers in a specific order your chance is smaller than when the order doesn't matter c. No, both events are equally likely to occur d. This depends on what the other players in the game throw

Figure 3-6. Post-test item measuring conceptual knowledge

Procedural knowledge is "the ability to execute action sequences to solve problems" (Rittle-Johnson et al., 2001, p. 346). The post-test contained 9 open-ended items aiming at measuring procedural knowledge (see Figure 3-7).

There is a man at a fair who says he will predict the 2 months in which you and your companion were born. The man does not have to specify who was born in which month. When he correctly predicts both months, he wins the stake; when his prediction is not correct, you win the stake and can choose a cuddly toy. You and your friend decide to take the chance. You were born in July and your friend was born in May. What is the chance that the man correctly predicts these months and wins?

Figure 3-7. Post-test item measuring procedural knowledge

The correct answers to the items presented in Figure 3-5, Figure 3-6, and Figure 3-7, are respectively: answer C; answer B; and (2/12)*(1/12)=1/72.

2.7 Procedure

The experiment was carried out in a real school setting in three sessions, each separated by a one-week interval. Students worked individually and they were told that they could work at their own pace.

The first session started with some background information with regard to the experiment (general purpose of the research, the domain of interest, learning goals, etc.). This was followed by the pre-test. It was announced that the post-test would contain more items of greater difficulty than the pre-test, but that the pre-test items nonetheless would give an indication of what kind of items to expect on the post-test. At the end of the pre-test the students received a short, printed introductory text introducing the domain. The first session was limited to 50 minutes in duration. During the last 15 minutes of the session, the use of the learning environment was demonstrated. Use of the representational tools was demonstrated for students in the experimental conditions. They were informed of the beneficial effects on learning of constructing representations and they were told that they could use the tool any time they wanted while working in the learning environment. During the second session, students worked with the learning environment; students in the experimental conditions were encouraged to use the representational tool to construct a domain representation while working with the learning environment. Again they were informed of the beneficial effects of constructing representations and they were told that they could use the tool any time they wanted while working in the learning environment. The duration of this session was set at 70 minutes. Although it was possible to take a non-linear path through the learning environment, students were advised to go through the sections in order because they build upon each other. While working with the representational tool, some students asked the experimenter if the quality of the representation they constructed would count as well for their final grade. They were told that it was very important to use the tool, that the constructed representation could possibly play some role in determining the grade, but that it in any case would be very helpful for preparing oneself for the post-test.

The third session was set at 50 minutes. First, students were allowed to use the learning environment for 10 minutes in order to refresh their memories with regard to the domain. Then all students had to close their domain representations and learning environments, and had to complete the post-test. When students finished the test they were allowed to leave the classroom.

2.8 Data preparation

The domain representations constructed by the students were scored using a scoring rubric (see Appendix). This rubric revolved around the principle that scoring of the domain representation should not be biased by the representational format of the representational tool, that is, all types of representations should be scored on the basis of exactly the same criteria. The maximum possible score was 8 points. The rubric was used to assess whether domain representations reflected the concepts of replacement and order, presented calculations, referred to the concept of probability, and indicated the effects of size of (sub)sets, replacement, and order on probability.

3 RESULTS

3.1 Use of representational tools

The first aspect of our research question was whether representational formats have differential effects on the likelihood that students engage in constructing representations. Of the 33 students provided with a conceptual representational tool, 17 students (52 percent) created a domain representation. This was about the same for students provided with a textual representational tool: 15 of the 32 students (47 percent) constructed a representation. The arithmetical tool turned out to be used the least: 6 out of 30 students (20 percent) used the tool. A Chi-Square analysis showed these differences between conditions were significant, $X^2(2, N = 95) = 7.45$, $p < .05$. The data show that the arithmetical format is clearly less ready-to-hand for creating external representations than the other two formats. The next question concerns the quality of the representations created by the students.

In Table 3-1 the average quality scores of the constructed representations as these were assessed with the scoring protocol are displayed. All representations were scored by two raters who worked independently. The inter-rater agreement was .89 (Cohen's Kappa).

Table 3-1
Quality scores of constructed representations

	Representational format											
	Conceptual (*n*=17)				Arithmetical (*n*=6)				Textual (*n*=15)			
	M	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*
Score	2.4	1.0	1	5	2.7	2.0	1	6	2.7	1.0	1	4

A one-way ANOVA showed that the format in which a representation was constructed did not influence its quality, $F(2,37) = 0.42$, MSE = 0.62, $p = .66$.

3.2 Time-on-task

The log files provided data about how much time students spent on the learning task (see Table 3-2).

Table 3-2
Time-on-task (min.)

	Condition							
	Conceptual (*n*=33)		Arithmetical (*n*=30)		Textual (*n*=32)		Control (*n*=32)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Total time	69.64	13.95	66.95	17.61	66.64	18.32	65.48	15.57
Tool-use	70.84	14.17	70.62	15.98	70.50	16.85		
No-tool-use	68.38	14.05	66.04	18.19	63.23	19.38		

The data presented in Table 3-2 were analyzed by means of 3 x 2 ANOVA with experimental conditions (Conceptual, Arithmetical, and Textual) and tool-use as factors. Subsequently, these data are compared to the control condition. With regard to time on task, no differences were observed between conditions ($F(2,89) = 0.22$, MSE = 60.89, $p = .81$), tool-use ($F(1,89) = 1.60$, MSE = 449.92, $p = .21$), and no interaction was observed ($F(2,89) = 0.17$, MSE = 46.99, $p = .85$). A one-way ANOVA in which the tool-users from each experimental condition and the students from the control condition were included, showed that tool-users and students in the control condition spent the same amount of time on the learning task, $F(3,66) = 0.65$, MSE = 156.48, $p = .59$. The same was true for no-tool-users and students in the control condition, they also spent the same amount of time on the task, $F(3,85) = 0.26$, MSE = 74.19, $p = .85$.

3.3 Knowledge measures

Two measures of knowledge were obtained: prior knowledge (pre-test score), and post-test score. The reliability, Cronbach's α, was α = .40 for the pre-test and α = .80 for the post-test. The pre-test reliability was rather low, but sufficient for the purpose of verifying if students did not have too much prior knowledge and that there were no differences between conditions. The scores on the knowledge measures are displayed in Table 3-3. In this table and the subsequent analyses a distinction is made for the three experimental conditions between students who used the representational tool and those who did not.

Table 3-3
Knowledge measures

Knowledge type (max. nr. of items)	Condition							
	Conceptual (n=33)		Arithmetical (n=30)		Textual (n=32)		Control (n=32)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
MathGrade(10)	6.46	1.61	5.89	1.55	6.25	1.54	6.27	1.64
Tool-use	7.12	1.52	6.62	1.63	6.73	1.37		
No-tool-use	5.75	1.41	5.71	1.51	5.82	1.58		
PRE-TEST (12)	5.70	1.36	5.43	1.85	5.25	1.59	5.47	1.95
Tool-use	5.94	1.39	5.67	1.75	5.40	1.60		
No-tool-use	5.44	1.32	5.38	1.91	5.12	1.62		
POST-TEST								
Concept. (12)	5.70	1.36	5.43	1.85	5.25	1.59	5.47	1.95
Tool-use	5.94	1.39	5.67	1.75	5.40	1.60		
No-tool-use	5.44	1.32	5.38	1.91	5.12	1.62		
Procedur. (10)	4.15	2.32	3.67	2.70	3.66	2.04	3.75	2.57
Tool-use	4.82	2.33	5.17	3.19	3.87	1.96		
No-tool-use	3.44	2.16	3.29	2.49	3.47	2.15		
Situational (4)	2.94	1.25	2.87	1.33	2.66	1.31	2.63	1.36
Tool-use	3.41	1.00	3.67	0.82	3.07	1.22		
No-tool-use	2.44	1.32	2.67	1.37	2.29	1.31		
OVERALL (26)	16.30	4.51	16.30	4.11	15.59	3.98	15.44	4.68
Tool-use	18.00	4.27	19.17	3.82	16.73	3.49		
No-tool-use	14.50	4.15	15.58	3.93	14.59	4.21		

The data presented in Table 3-3 were analyzed by means of 3 x 2 ANOVAs with experimental conditions (Conceptual, Arithmetical, and Textual) and tool-use as factors. After that, a separate analysis that compares the data of the experimental groups with the control group is presented.

Students were asked for their latest school report grade in mathematics. This grade, which can range from 1 (very, very poor) to 10 (outstanding), was interpreted as an indication of the student's general mathematics achievement level. An ANOVA showed that there were no differences between conditions with regard to math grade, $F(2,89) = 0.22$, MSE = 0.49, $p = .80$. With regard to tool-use a difference was found: students who constructed representations (tool-use) in general had somewhat higher math grades than students who did not use their representational tool to construct a representation, $F(1,89) = 10.01$, MSE = 22.38, $p < .01$. No interaction between condition and tool-use was found, $F(2,89) = 0.24$, MSE = 0.54, $p = .79$.

With regard to prior knowledge (pre-test score) no differences were observed between conditions, $F(2,89) = 0.58$, MSE = 1.52, $p = .56$, tool-use, $F(1,89) = 0.97$, MSE = 2.55, $p = .33$, and no interaction was observed, $F(2,89) = 0.05$, MSE = 0.12, $p = .96$.

Math grade was entered as a covariate in the analysis of learning outcomes. With respect to overall post-test scores, no differences were observed between conditions, $F(2,88) = 1.39$, MSE = 20.23, $p = .26$. A main effect of tool-use was found: students who constructed a domain representation showed significantly higher overall post-test scores, $F(1,88) = 5.65$, MSE = 82.41, $p < .05$, $\eta_p^2 = .06$. No interaction effects were observed between condition and tool-use, $F(2,88) = 0.24$, MSE = 3.53, $p = .79$.

With regard to conceptual knowledge, no main effect of condition ($F(2,88) = 1.58$, MSE = 4.46, $p = .21$), tool-use ($F(1,88) = 3.54$, MSE = 9.99, $p = .06$), or interaction between the two was found ($F(2,88) = 0.07$, MSE = 0.19, $p = .94$).

In the case of procedural knowledge no main effects were observed for condition ($F(2,88) = 0.59$, MSE = 2.58, $p = .56$) and tool-use ($F(1,88) = 0.96$, MSE = 4.20, $p = .33$), and no interaction effect was found ($F(2,88) = 0.76$, MSE = 3.31, $p = .47$).

Regarding situational knowledge, condition did not play a significant role ($F(2,88) = 0.93$, MSE = 1.45, $p = .10$), but tool-use did, $F(1,88) = 9.60$, MSE = 14.97, $p < .01$, $\eta_p^2 = 0.10$. No interaction effect was observed, $F(2,88) = 0.07$, MSE = 0.11, $p = .93$.

Finally, Pearson product-moment correlation coefficients between the quality of the constructed representations (in general, but also for each format) and knowledge type scores were calculated. No correlations were observed except that between the quality of representations in general (regardless of format) and procedural knowledge ($r = .33$, $p < .05$).

It could be argued that students who use the representational tools are more compliant, interested, and motivated and that students who don't use the tools are the less compliant, interested and motivated students. Differences between these groups as they were found could then be attributed to these characteristics and not to the factor tool use. A comparison of the data from the experimental groups with the data from the control group makes this assumption highly unlikely. Students in the control condition came from the same population as the students in the experimental conditions. They formed a cross-section of both the group of students who chose not to use the tool and the group who did use the tool. The performance of the control group can therefore be considered average.

The supposedly less motivated, compliant, and/or self-regulated no-tool-users are expected to perform below average, the tool-users to perform above average. However, the data do not confirm this expectation as can be observed in Figure 3-8.

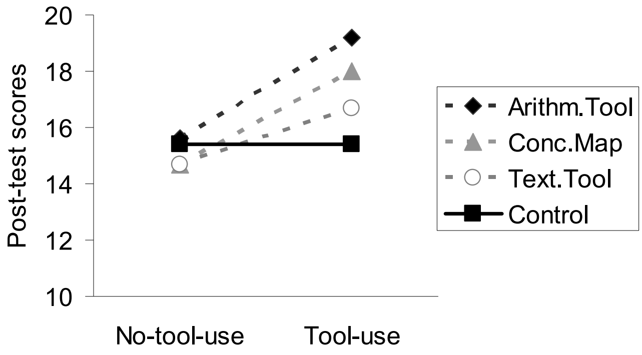


Figure 3-8. Post-test scores of experimental conditions and control condition

Figure 3-8 indicates that the learning outcomes of students who did not use a representational tool are equal to learning outcomes in the control condition. A statistical comparison of the overall post-test scores of the no-tool-use group and the control condition confirmed this picture, $t(87) = -0.48$, $p = .63$. Furthermore, it was established that the no-tool-use group and the control condition obtained equal scores with regard to conceptual knowledge ($t(87) = 0.10$, $p = .92$), procedural knowledge ($t(87) = -0.69$, $p = .49$), and situational knowledge ($t(87) = -0.45$, $p = .65$). Therefore, in terms of learning results the no-tool-use group and the control condition perform equal.

When the tool-use group and the control condition are compared with each other, it is found that with regard to overall post-test scores the tool-use group outperformed the control condition, $t(68) = 2.19$, $p < .05$, Cohen's $d = .52$. Both obtained comparable scores on conceptual knowledge ($t(57.45) = 1.96$, $p = .05$) and procedural knowledge ($t(68) = 1.28$, $p = .21$). With regard to situational knowledge, the tool-use group outperformed the control condition, $t(58.32) = 2.33$, $p < .05$, Cohen's $d = .57$.

4 DISCUSSION AND CONCLUSION

Constructing a representation of a domain is thought to be beneficial for learning and understanding (Cox, 1999). The format in which such a representation is constructed is assumed to have differential effects on knowledge construction and domain understanding. In the current study three formats for constructing representations in the domain of combinatorics and probability theory have been compared in a between-subjects pre-test post-test experiment with three experimental conditions and one control condition. The three experimental conditions were provided with a simulation-based inquiry learning environment that was equipped with a representational tool, that is, a support tool that could be used by students to construct a domain representation. The experimental conditions differed with respect to the format of the tool, that could be conceptual (i.e., a concept map), arithmetical, or textual. The following questions guided this study. Do representational formats have differential effects on the likelihood that students use the support and engage in constructing representations? Does format have differential effects on the quality of the representations students construct? Does the construction of a representation of a domain lead to better learning outcomes than not constructing a representation? And, if students construct a representation, does format have differential effects on domain understanding? In order to gain a more full understanding of the effects of support in the form of representational tools, the data of the experimental groups were also compared with students in a Control conditions using the same learning environment as used in the experimental conditions, but without a representational tool.

The findings show that more representational tools with a conceptual or a textual format are more readily used than a tool with an arithmetical format. For the target population the conceptual or textual formats apparently afford the construction of a representation more than a conventional arithmetical format. This finding to some extent corroborates the observation by Tarr and Lannin (2005) that students initially avoid using conventional ways of representing probabilities, using instead alternative representational forms. It was also observed that formats do not have a differential effect on the quality of the constructed representations. The post-test scores show that there is no direct relation between the format of the domain representation and learning outcomes in terms of conceptual, procedural, or situational knowledge. It was found, however, that the construction of a domain representation in general is related to higher post-test scores. Furthermore, it was found that constructing representations, regardless of the format, is associated with significantly higher levels of situational knowledge. This type of knowledge is a prerequisite for going beyond the superficial details of problems in order to recognize the concepts and structures that underlie the problem and to decide which operations are required to solve it (e.g., Fuchs et al., 2004). In the case of probability instruction, the approach that needs to be taken to solve a problem is very dependent on the correct classification of the problem (Lipson et al., 2003). The differences could not be attributed to time-on-task. Students in all conditions and regardless whether or not they constructed a domain representation, all spent the same amount of time on their learning task.

In the introduction section it was discussed that students often have a hard time understanding the domain of combinatorics and probability theory. The question was raised as to what could be done to improve instruction about this domain. In a previous study it was shown that the representational format in which the domain is *presented* to the students affects learning outcomes (Kolloffel et al., in press). What the current study adds to the understanding of how instruction in this domain can be improved is that *creating* a representation of the domain can also be beneficial for learning outcomes. The format used to create this representation is found to play a critical but indirect role. Although format affects neither the quality of the representation nor the learning outcomes, it does influence the likelihood that students engage in constructing a representation. The activity of constructing a domain representation is primarily associated with becoming more knowledgeable about the problem categories in this domain so as to identify these categories in problem statements. Choosing the right format for constructing a domain representation is a useful first step towards promoting students to externalize their knowledge and to benefit from the effects of this type of learning support. The findings also suggest that it is worthwhile to search for more ways in which more students will engage in constructing representations. In other studies of representational tool use (Gijlers & de Jong, submitted; Gijlers, Saab, van Joolingen, van Hout-Wolters, & de Jong, submitted; Suthers & Hundhausen, 2003; van Drie et al., 2005), students worked collaboratively. In future studies of improving instruction of combinatorics and probability theory, it can be useful to examine the added value of collaborative learning in combination with representational tools.

Chapter 4

Do representational tools support understanding in individual and collaborative learning?[1]

Abstract

The aim of the current study was to examine the effects of providing learning support in the form of representational tools, that is, tools students can use to construct domain representations (e.g., concept maps). Three different formats of representational tools (conceptual, arithmetical, or textual) were compared and the following questions guided the study. First, are students inclined to use a representational tool and does the format of a tool affect this inclination? Second, does using a representational tool lead to better learning outcomes and are there differences in effectiveness between different formats? Data were collected in a collaborative learning setting (61 pairs) and were compared to data collected in an earlier study in which an individual learning setting was used (95 individuals). In both studies a between-subjects pre-test post-test design was applied in which individual or paired students were randomly distributed over three conditions (conceptual, arithmetical and textual format of the representational tool). All students were provided with a simulation-based learning environment about combinatorics and probability theory equipped with a representational tool. It was found that format clearly influenced students' inclination to use a tool, with a higher use (around 50%) of the textual and conceptual tools compared to the arithmetical format (around 20% use). In the collaborative setting, neither using the support nor the tool format did affect learning outcomes. In general, the collaborative students obtained better learning outcomes compared to individuals, but if individuals used the support, their learning outcomes equaled those of students in the collaborative setting.

[1] This chapter is adapted from Kolloffel, B., Eysink, T. H. S., & de Jong, T. (in preparation). *Do representational tools support understanding in combinatorics instruction? Comparing effects in collaborative and individual learning settings.*

1 INTRODUCTION

External representations are known to play a critical role in learning and understanding. The properties of representational formats influence which information is attended to and how people tend to seek, organize and interpret information (e.g., Ainsworth & Loizou, 2003; Cheng, 1999; Larkin & Simon, 1987; Zhang, 1997). In pictures or diagrams for example, information about topological and structural relations is explicit, allowing people to draw inferences easier and quicker compared to textual representations (Larkin & Simon, 1987). External representations can be presented to students, but students can also construct representations themselves. Cox (1999) argues that the process of constructing a representation helps students to refine and disambiguate their domain knowledge because it elicits self-explanation effects and consists of dynamic iterations and interactions between the constructed representations and mental representations, which can make students aware of unnoticed gaps and/or ambiguities in their mental representations. Examples of activities in which students construct a representation are: writing a summary (Foos, 1995; Hidi & Anderson, 1986), creating a drawing (Van Meter et al., 2006; Van Meter & Garner, 2005), building a runnable computer model (Löhner et al., 2003; Manlove et al., 2006), or constructing a concept map (Nesbit & Adesope, 2006; Novak, 1990, 2002). Kolloffel, Eysink, and de Jong (submitted) studied the effects of representational tools, that is, tools students can use to construct domain representations (e.g., a concept mapping tool). The study was performed using a subdomain of mathematics that is known to be notoriously difficult: combinatorics and probability theory. Three different formats of representational tools were tested. A *conceptual representational tool* (i.e., a concept mapping tool. See Figure 4-2 in the Method section) could be used to create a concept map of the domain. An *arithmetical representational tool* (see Figure 4-3 in the Method section) enabled students to construct arithmetical representations (e.g., formulas, equations). A *textual representational tool* (see Figure 4-4 in the Method section) resembled simple word processing software. Each of the tools was integrated in a simulation-based inquiry learning environment. It was found that students who constructed representations showed significantly higher post-test scores, and they also showed enhanced levels of situational knowledge, which is a prerequisite for going beyond the superficial details of problems. Furthermore, when students were provided with a conceptual or textual representational tool they were much more likely to construct representations than when provided with a representational tool with an arithmetical format.

Also in collaborative learning, external representations and their format may play a crucial role in determining the effectiveness of the learning environment. In addition to beneficial effects of the construction of a representation of the domain per se, in collaborative learning an external representation may form the pivot around which students share and discuss knowledge. Suthers and Hundhausen (2003), for example, found that the focus of students' discourse and collaborative activities were influenced by the format in which students had to construct a representation. They compared three types of representational tools (concept maps, evidence matrix, and text) in a task in which pairs of students investigated complex science and public health problems. The representational tools were integrated in an electronic learning environment in which students explored a sequence of information pages. As the students worked through these pages, they could use the representational tool to record information. It was found that pairs using an evidence matrix representation discussed and represented issues of evidence more than pairs using other representations. Second, pairs using visually structured representations (concept map, evidence matrix) revisited previously discussed ideas more often than pairs using text. Third, it was observed that the evidence matrix not only prompted novices to consider relevant relationships, but made them spend considerable time and resources on irrelevant issues as well.

In another study, van Drie, van Boxtel, Jaspers, and Kanselaar (2005) compared three different formats of representational tools in a historical writing task in a computer-supported collaborative learning (CSCL) environment. They compared argumentative diagrams, lists, and matrices. These representational tools were integrated in the learning environment, and besides a chat tool, the representational tools turned out to be a tool through which the (physically separated) students communicated with each other. It was found that matrices consisting of a table format that could be filled in by the students, supported domain-specific reasoning and listing arguments, whereas argumentative diagrams, organizing and linking arguments in a two-dimensional graphical way, made students focus more on the balance between pro and contra arguments.

Both studies demonstrated significant effects of representational format on collaboration *processes*. However, neither of the studies found effects on learning *outcomes*. In the van Drie et al. (2005) study, a control condition in which students learned collaboratively but did not have to construct an external representation, scored equally well on the post-test compared to students in the experimental conditions. Suthers and Hundhausen (2003) also did not find effects on learning outcomes.

Some studies in collaborative learning settings do report effects of representational tools on learning outcomes however. For example, Gijlers and de Jong (submitted) studied the effects of a shared concept mapping tool in a collaborative simulation-based inquiry learning task on one dimensional kinematics. In the experimental condition students were provided with a concept mapping tool during the learning task, whereas students in the control condition learned without a concept mapping tool. The findings showed that students in the concept mapping condition performed significantly better on an essay test and on an intuitive knowledge test. Intuitive knowledge is considered to be a quality of conceptual knowledge that taps on understanding how changes of one variable affect other variables. This knowledge is often hard to verbalize (Swaak & de Jong, 1996). Gijlers and de Jong (submitted) observed that in the concept mapping condition the intuitive knowledge scores were significantly and positively related to the percentage of chat messages related to conclusion and interpretation. The latter is in line with some other studies in which it was found that intuitive knowledge is particularly fostered by processes of interpretation and sense-making (Reid, Zhang, & Chen, 2003; Zhang, Chen, Sun, & Reid, 2004).

The different studies suggest that representational tools in collaborative learning sometimes affect learning outcomes. Possibly, this depends to some extent on whether the post-test is sensitive to effects of tools and their formats on learning (e.g., does the post-test measure intuitive knowledge). The study by Gijlers and de Jong (submitted) revealed effects of a representational tool on learning outcomes, but they did not compare different formats of tools. Suthers and Hundhausen (2003) and van Drie et al. (2005) compared different formats but did not find effects on learning outcomes, possibly because the post-tests used did not detect effects of constructing representations (e.g., intuitive knowledge).

The aim of the current study was to examine the effects of providing support in the form of representational tools in collaborative learning, taking into account differential effects of format and possible effects on intuitive knowledge. The experiment in which Kolloffel et al. (submitted) examined the effects and affordances of representational tools and their formats in an individual learning setting was taken as point of departure. This study was repeated in a collaborative learning setting, which also allows the data from the collaborative setting to be compared to the data from the individual learning setting. Both studies differed in learning setting, collaborative vs. individual learning, but shared the following research questions. First, are students inclined to use a representational tool and does the format of a tool affect this inclination? Second, does using a representational tool lead to better learning outcomes and are there differences in effectiveness between different formats?

2 METHOD

2.1 Participants

The data collection of the study in the collaborative setting took place exactly one year after the study with individual students at the same school. In the *collaborative learning study*, in total 128 secondary education students entered the experiment. In total, the data of 61 pairs could be analyzed. The average age of these 56 boys and 66 girls was 14.62 years (SD =.57). In the *individual learning study*, 95 secondary education students, 50 boys and 45 girls, participated (see Kolloffel et al., submitted). The average age of the students was 14.62 years (SD =.63).

The domain of combinatorics and probability theory was part of the regular curriculum and both experiments took place a couple of weeks before this subject would be treated in the classroom. The students attended the experiment during regular school time; therefore, participation was obligatory. They received a grade based on their post-test performance.

The experiments employed a between-subjects design with the format of the provided representational tool (conceptual, arithmetical, or textual) as the independent variable. Students were randomly assigned to conditions. Of the 61 pairs in the collaborative setting, 22 pairs were in the Conceptual condition, 19 pairs in the Arithmetical condition, and 20 pairs in the Textual condition. Of the 95 students in the individual learning setting, 33 were in the Conceptual condition, 30 in the Arithmetical condition, and 32 in the Textual condition.

2.2 Domain

The domain of instruction was combinatorics and probability theory. An example of a problem in this domain is: what is the probability that a thief will guess the 4-digit PIN-code of your credit card correctly in one go? The essence of combinatorics is determining how many different combinations can be made with a certain set or subset of elements. In order to determine the number of possible combinations, one also needs to know 1) whether elements may occur repeatedly in a combination (replacement) and 2) whether the order of elements in a combination is of interest (order). On basis of these two criteria, four so-called problem categories can be distinguished. The PIN-code example matches category "replacement; order important". When the number of possible combinations is known, the probability that one or more combinations will occur in a random experiment can be determined.

2.3 Learning environment

The instructional approach used in this study is based on inquiry learning (de Jong, 2005, 2006). Computer-based simulation is a technology that is particularly suited for inquiry learning. Computer-based simulations contain a model of a system or a process. By manipulating the input variables and observing the resulting changes in output values the student is enabled to induce the concepts and principles underlying the model (de Jong & van Joolingen, 1998).

The learning environment used in the current study, called Probe-XMT (see Figure 4-1), was created with SIMQUEST authoring software (van Joolingen & de Jong, 2003).

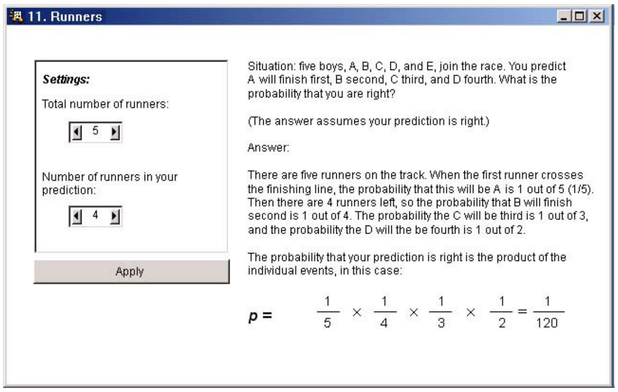


Figure 4-1. Screen dump Probe-XMT simulation

In the box on the left-hand side of the simulation (see Figure 1), students could manipulate input variables. On the right-hand side of the simulation the resulting effects of the manipulations on the output values could be observed. In this case the output consisted of a text and an equation that changed whenever the input variables were changed. Probe-XMT consisted of five sections. Four of these sections were devoted to each of the four problem categories within the domain of combinatorics. The fifth section aimed at integrating these four problem categories. Each section used a different cover story, that is, an everyday life example of a situation in which combinatorics and probability played a role. Each cover story exemplified the problem category treated in that section. In the fifth (integration) section, the cover story applied to all problem categories.

Each of the five sections contained a series of questions (both open-ended and multiple-choice items), all based on the cover story for that particular section. These questions involved determining which problem category matched the given cover story (situational knowledge), calculating the probability in a given situation (procedural knowledge), and selecting a description that matched the relation between variables most accurately (conceptual knowledge). In the case of the multiple-choice items, the students received feedback from the system about the correctness of their answer. If the answer was wrong, the system offered hints about what was wrong with the answer. Students then had the opportunity to select another answer. In the case of the open-ended questions, students received the correct answer after completing and closing the question.

Most of the questions were accompanied by simulations that could be used to explore the relations between variables within the problem category. In the simulations, students could manipulate variables and observe the effects of their manipulations on other variables. The simulations used a combination of textual and arithmetical representations. This combination of representations was found to have benefits in terms of learning outcomes and mental effort (Kolloffel et al., in press).

The learning environment automatically registered user actions. User actions that were logged included measures like user path through the learning environment (which parts of the learning environment were opened, when, for how long, and in what sequence) and the number and nature of manipulations carried out in the simulations (how many experiments were carried out and the input values of each experiment).

2.4 Representational tools

Students were encouraged to construct a representation of the domain. This representation could be used to summarize principles underlying the domain, the variables playing a role in the domain, and their mutual relationships. Students could create their representations by means of an electronic on-screen representational tool. There were three types of representational tools, one for each experimental condition: (a) a conceptual representational tool, (b) an arithmetical representational tool, and (c) a textual representational tool. The *conceptual representational tool* (see Figure 4-2) could be used to create a concept map of the domain. Students could draw circles representing domain concepts and variables. Keywords could be entered in the circles. The circles could be connected to each other by arrows indicating relations between concepts and variables. The nature of these relations could be specified by attaching labels to the arrows.

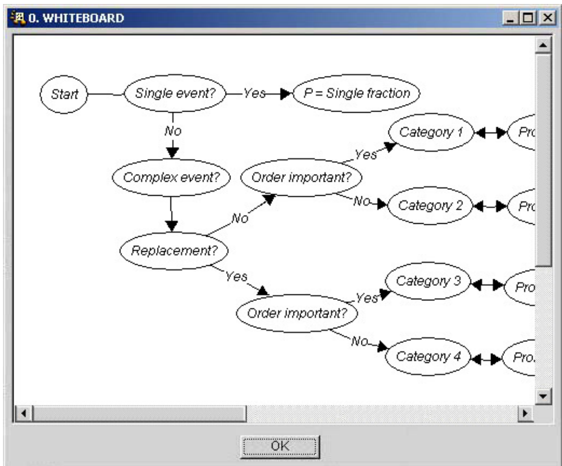


Figure 4-2. Conceptual representational tool

In the *arithmetical representational tool* (see Figure 4-3), students could use variable names, numerical data, and mathematical operators (division signs, equation signs, multiplication signs, and so on) in order to express their knowledge.

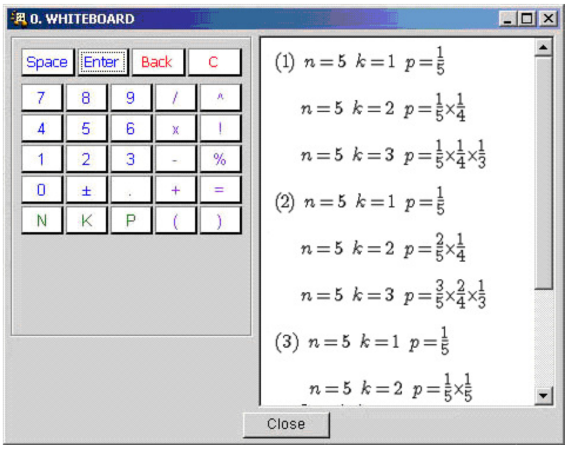


Figure 4-3. Arithmetical representational tool

Finally, the *textual representational tool* (see Figure 4-4) resembled simple word processing software, allowing textual and numerical input.

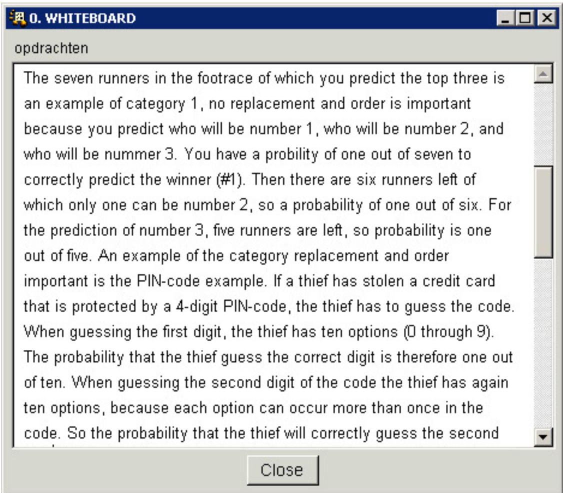


Figure 4-4. Textual representational tool

In the current study, the effects of representational tools were tested outside the lab, in real class room settings. The tools were intended as means to *support* students while learning, not as means to *assess* learning. Assessment is mostly obligatory in classroom settings, whereas making use of support is not. For reasons of ecological validity, the use of the representational tools was therefore not obligatory, although students were strongly advised to use the tool and they were informed that using the tool would help them to better prepare themselves for the post-test.

2.5 Knowledge measures

Two knowledge tests were used in this experiment: a pre-test and a post-test. The tests contained 12 and 26 items respectively. The pre-test aimed at measuring (possible differences in) the prior knowledge of the students. The post-test was specifically designed to measure the effects of external representations on learning outcomes. The sensitivity and reliability of the test items have been established in many studies performed across Germany and The Netherlands in the last couple of years (see e.g., Berthold & Renkl, in press; Eysink et al., submitted; Gerjets, Scheiter, Opfermann, Hesse, & Eysink, in press; Kolloffel et al., in press; Wouters, Paas, & Van Merriënboer, 2007). The post-test consisted of different types of items. Sweller (1989, p. 458) defined a mathematical schema as "a cognitive construct that permits problem solvers to recognize problems as belonging to a particular category requiring particular moves for solution". A complete schema therefore rests on three pillars: conceptual knowledge, procedural knowledge, and situational knowledge. Conceptual knowledge is "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson et al., 2001, p.346). Conceptual knowledge develops by establishing relationships between pieces of information or between existing knowledge and new information. The post-test contained 12 multiple choice items aiming at measuring conceptual knowledge. Four of these items were intended to measure regular conceptual knowledge (see Figure 4-5 for an example).

You have a deck of cards from which you select 4 cards. You predict that you will select an ace, a king, a queen and a jack in this specific order. Does it matter whether you put back the selected cards before each new selection or not?

a. Yes, your chances increase when you put back the selected cards
b. Yes, your chances decrease when you put back the selected cards
c. No, your chances remain the same whether you put back the selected cards or not
d. This depends on whether the deck of cards is complete or not

Figure 4-5. Post-test item measuring conceptual knowledge

Eight items were intended to measure intuitive conceptual knowledge (see Figure 4-6 for an example). Items measuring conceptual knowledge and intuitive conceptual knowledge differed in three respects (Eysink et al., submitted): first, the situation described in the problem statement regarding the intuitive items was the same for each item and was presented prior to the items instead of being presented with each separate item; second, the intuitive items offered two alternatives instead of four; finally, students were asked to answer the intuitive items as quickly as possible, as intuitive knowledge is characterized by a quick perception of meaningful situations (Swaak & de Jong, 1996).

(Answer the following question(s) as quickly as possible)

There are a number of marbles in a bowl. Each marble has a different color. You will pick at random (e.g., blindfolded) a number of marbles from the bowl, but before you do you predict which colors you will pick.

The chance your prediction proves to be correct is higher in case of:

a. No replacement; order not important
b. Replacement; order important

Figure 4-6. Post-test item measuring intuitive conceptual knowledge

Procedural knowledge is "the ability to execute action sequences to solve problems" (Rittle-Johnson et al., 2001, p.346). The post-test contained 10 open-ended items aiming at measuring procedural knowledge (see Figure 4-7 for an example).

In a pop music magazine you see an ad in the rubric FOR SALE in which a ticket for a spectacular concert of your favorite pop group is offered. Unfortunately the last 2 digits of the telephone number, where you can obtain information about the ticket, are not readable anymore. You really like to have the ticket and decide to choose the 2 digits randomly. What is the probability that you dial the correct digits on your first trial?

Figure 4-7. Post-test item measuring procedural knowledge

Situational knowledge (de Jong & Ferguson-Hessler, 1996) enables students to analyze, identify, and classify a problem, to recognize the concepts that underlie the problem, and to decide which operations need to be performed to solve the problem. Four multiple-choice items were included in the post-test to measure this type of knowledge (see Figure 3-5 for an example).

You throw a dice 3 times and you predict that you will throw 6-4-2 in that order. What is the characterization of this problem?

a. order important; replacement
b. order important; no replacement
c. order not important; replacement
d. order not important; no replacement

Figure 4-8. Post-test item measuring situational knowledge

The correct answers to the items presented in Figure 4-5, Figure 4-6, Figure 4-7, and Figure 4-8, are respectively: answer B; answer A; (1/10)*(1/10)=1/100; and answer A.

2.6 Procedure

The experiments were carried out in a real school setting in three sessions, each separated by a one-week interval. The procedures in both the individual and collaborative setting were identical.

The first session started with presenting students some background information with regard to the experiment (general purpose of the study, the domain of interest, learning goals, and so on). This was followed by the pre-test. In both the individual and the collaborative setting, students completed the pre-test individually. It was announced that the post-test would contain more items of greater difficulty than the pre-test, but that the pre-test items nonetheless would give an indication of what kind of items to expect on the post-test. At the end of the pre-test the students received a printed introductory text in which the domain was introduced. The duration of the first session was limited to 50 minutes. During the last 15 minutes of the session, the students received an explanation of how their representational tool could be operated and they could practice with the tool.

During the second session, students worked with the learning environment and had to construct a domain representation using the representational tool. The duration of this session was set at 70 minutes. Students in the individual learning setting worked alone. In the collaborative learning setting students were allowed to choose their partner themselves. Communication between students was on a face-to-face basis: the collaborating students were sitting next to each other, using the same computer terminal. Despite the possibility of following a non-linear path through the learning environment, students were advised to keep to the order of sections because they build upon each other.

The third session was set at 50 minutes. First, students were allowed to use the learning environment for 10 minutes in order to refresh their memories with regard to the domain. Then all students had to close their domain representations and learning environments, and had to complete the post-test. In both the individual and the collaborative setting, students completed the post-test individually.

2.7 Data preparation

The domain representations constructed by the students were scored by means of a scoring rubric (see appendix). This rubric revolved around the principle that scoring of the domain representation should not be biased by the representational format of the representational tool, that is, all types of representations should be scored on the basis of exactly the same criteria. The maximum number of points that could be assigned on the basis of the rubric was 8 points. The rubric was used to assess whether domain representations reflected the concepts of replacement and order, presented calculations, referred to the concept of probability, indicated the effect of size of (sub)sets on probability, and the effects of replacement and order on probability.

3 RESULTS

3.1 Use of representational tools

The percentages of students in each condition who used the representational tool to construct a domain representation are displayed in Figure 4-9.

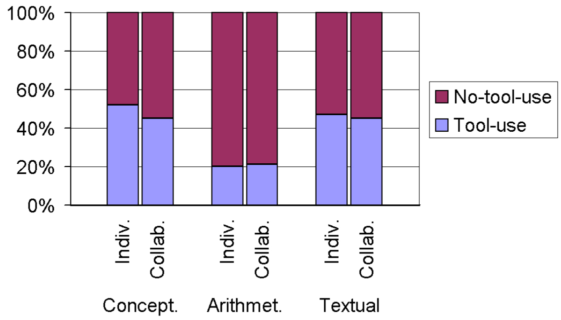


Figure 4-9. Percentage of students in each condition who did or did not construct a representation

When provided with a *conceptual tool*, 52 percent of the individual students and 45 percent of the pairs of students used it. A Chi-Square analysis showed that these percentages can be considered equal, that is do not differ significantly, $X^2(1, N = 55) = 0.19$, *n.s.*

Of students provided with an *arithmetical tool*, 20 percent of the individuals and 21 percent of the pairs used it, which is no significant difference, $X^2(1, N = 49) = 0.01$, *n.s.*

When provided with a *textual tool*, 47 percent of the individuals and 45 percent of the pairs of students used it, which again is no significant difference, $X^2(1, N = 52) = 0.02$, *n.s.*

As can be observed in Figure 4-9, the patterns of tool use are quite similar for the individual and the collaborative setting. The overall picture is that about 50 percent of the students provided with a conceptual or textual tool used the tool. Of students provided with the arithmetical tool, about 20 percent actually used the tool. A Chi-Square analysis showed that these differences between conditions are significant, $X^2(2, N = 156) = 10.58$, $p < .01$. Compared to students in the Arithmetical condition, students in the Conceptual condition used their tool more often ($X^2(1, N = 104) = 9.30$, $p < .01$) and so did students with a textual tool ($X^2(1, N = 101) = 7.49$, $p < .05$). No difference was observed between the Conceptual and the Textual condition ($X^2(1, N = 107) = 0.09$, *n.s.*).

In Table 4-1 the average quality scores of the constructed representations are displayed. In the case of representations constructed by pairs, the representations are considered a group product and therefore the quality scores are assigned to pairs and not to individuals. All representations were scored by two raters who worked independent from each other. The inter-rater agreement was .89 (Cohen's Kappa) for the individual setting and .92 for the collaborative setting.

Table 4-1
Quality scores of constructed representations

	Representational format											
	Conceptual (n=17)				Arithmetical (n=6)				Textual (n=15)			
	M	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*
Indiv	2.4	1.0	1	5	2.7	2.0	1	6	2.7	1.0	1	4
Collab	2.7	1.4	1	5	4.0	1.4	2	5	3.0	0.9	2	4

A two-way ANOVA with setting (individual vs. collaborative learning) and condition as factors showed that with regard to quality scores there was no main effect of setting ($F(1,55) = 3.69$, $p = .06$), no main effect of condition ($F(2,55) = 1.57$, $p = .22$), and no interaction effect ($F(2,55) = 0.71$, $p = .50$).

3.2 Time-on-task

The log files provided data about the amount of time students spent on the learning task (see Table 4-2).

Table 4-2
Time-on-task (min.)

	Condition							
	Conceptual (indiv. *n*=33; coll. 22 pairs)		Arithmetical (indiv. *n*=30; coll. 19 pairs)		Textual (indiv. *n*=32; coll. 20 pairs)		Total (indiv. *n*=95; coll. 61 pairs)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Individuals	69.64	13.95	66.95	17.61	66.64	18.32	67.78	16.57
Tool-use	70.84	14.17	70.62	15.98	70.50	16.85	70.67	15.13
No-tool-use	68.38	14.05	66.04	18.19	63.23	19.38	65.86	17.33
Collaborativ	62.36	4.98	60.90	8.41	65.91	14.30	63.07	9.95
Tool-use	64.88	3.98	65.68	2.16	63.13	5.08	64.34	4.19
No-tool-use	60.25	4.88	59.63	9.04	68.17	18.86	62.30	12.19

The data presented in Table 4-2 were analyzed by means of a three-way ANOVA with setting (individual vs. collaborative learning), condition, and tool-use as factors. Note that in the case of collaborative learning the process measures of the dyads were analyzed, not the measures of the individual students of the dyad.

With regard to time-on-task it was found that there was no main effect of setting ($F(1,144) = 3.18$, $p = .08$), condition ($F(2,144) = 0.03$, $p = .97$), or tool-use ($F(1,144) = 1.63$, $p = .20$). No interaction effects were observed.

3.3 Knowledge measures

Two measures of knowledge were obtained: prior knowledge (pre-test score), and post-test score. Both in the collaborative and the individual setting students completed the tests individually. The reliability, Cronbach's α, of the pre-test was .40 in the individual setting and .48 in the collaborative setting. The pre-test reliabilities were rather low, but sufficient for the purpose of verifying that students did not have too much prior knowledge and that there were no differences between settings and/or conditions. For the post-test the reliabilities of the individual and the collaborative setting were respectively: .80 and .78. Furthermore, students were asked for their latest school report grade in mathematics. This grade, which can range from 1 (very, very poor) to 10 (outstanding)) was interpreted as an indication of the student's general mathematics achievement level. It should be noted that this measure was reported by the students themselves and since no data from the school regarding math grades was available to the experimenters, the accuracy and reliability of the reported math grades should be considered with care. In Table 4-3 math grade and pre-test measures are presented.

Table 4-3
Math grade and pre-test measures

Knowledge type (max. nr. of items)	Condition: Conceptual (indiv. *n*=33) collab. *n*=44)		Arithmetical (indiv. *n*=30) collab. *n*=38)		Textual (indiv. *n*=32) collab. *n*=40)		Total (indiv. *n*=95) coll. *n*=122)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
M.GRADE (10)								
Indiv. (total)	6.46	1.61	5.89	1.55	6.25	1.54	6.21	1.57
Tool-use	7.12	1.52	6.62	1.63	6.73	1.37	6.89	1.46
No-tool-use	5.75	1.41	5.71	1.51	5.82	1.58	5.75	1.48
Collab. (total)	6.56	1.32	6.98	1.20	6.80	1.00	6.77	1.19
Tool-use	6.75	1.33	6.75	1.49	6.82	1.10	6.75	1.09
No-tool-use	6.40	1.32	7.04	1.14	6.79	0.93	6.84	0.94
PRE-TEST (12)								
Indiv. (total)	5.70	1.36	5.43	1.85	5.25	1.59	5.46	1.60
Tool-use	5.94	1.39	5.67	1.75	5.40	1.60	5.68	1.51
No-tool-use	5.44	1.32	5.38	1.91	5.12	1.62	5.32	1.65
Collab. (total)	5.70	1.94	5.71	1.52	6.35	1.88	5.92	1.81
Tool-use	5.90	1.89	6.00	1.51	5.67	1.65	5.83	1.70
No-tool-use	5.54	2.00	5.63	1.54	6.91	1.90	5.97	1.88

Three-way ANOVAs with setting (individual or collaborative), condition (Conceptual, Arithmetical, Textual), and tool-use (Tool-use or No-tool-use) as factors were performed to test for a priori differences with respect to math grade (general mathematics achievement level) and pre-test score (prior knowledge).

A difference regarding *math grade* was observed with respect to setting, $F(1,205) = 5.37$, $p < .05$, and tool-use, $F(1,205) = 6.97$, $p < .01$. No main effect of condition was found ($F(2,205) = 0.01$, $p = .99$). An interaction between setting and tool-use, $F(1,205) = 7.24$, $p < .01$, was observed. On average, the math grades of students in the collaborative learning setting were somewhat higher compared to the individual students. Furthermore, in the individual learning setting it was observed that students who used their representational tool had higher math grades compared to individuals who did not use the tool. The math grades of individuals who used the tool were equal to those of students in the collaborative setting.

With regard to *pre-test scores*, no main effects were found for setting ($F(1,205) = 3.12$, $p = .08$), condition ($F(2,205) = 0.06$, $p = .95$), or tool-use ($F(1,205) = 0.13$, $p = .72$). No interaction effects were observed either.

Therefore, in the analyses of post-test measures (see Table 4-4) only math grade was entered as a covariate.

Table 4-4
Post-test measures (corrected for math grade)

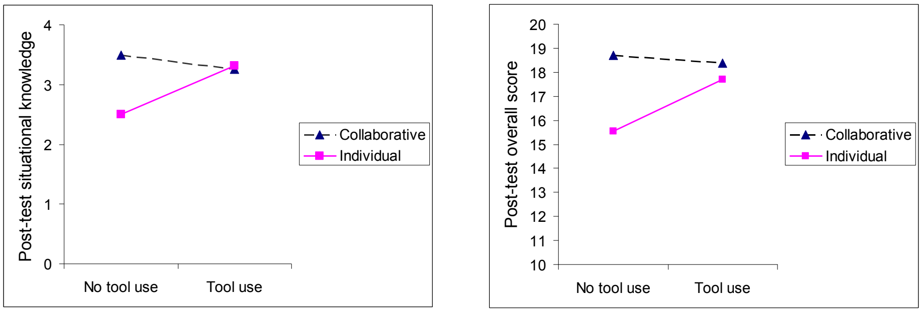
Knowledge type (max. nr. of items)	Conceptual (indiv. *n*=33) collab. *n*=44)		Arithmetical (indiv. *n*=30) collab. *n*=38)		Textual (indiv. *n*=32) collab. *n*=40)		Total (indiv. *n*=95) coll. *n*=122)	
	M	*SE*	*M*	*SE*	*M*	*SE*	*M*	*SE*
CONC. (12)								
Indiv. (total)	9.21	0.26	10.05	0.34	9.36	0.26	9.54	0.17
Tool-use	9.66	0.36	10.32	0.61	9.76	0.38	9.91	0.27
No-tool-use	8.77	0.38	9.78	0.31	8.95	0.36	9.17	0.21
Collab. (total)	11.32	0.23	10.79	0.30	10.64	0.24	10.92	0.15
Tool-use	11.41	0.33	10.71	0.53	10.57	0.35	10.90	0.24
No-tool-use	11.23	0.30	10.87	0.27	10.71	0.32	10.94	0.17
INTUIT (8)								
Indiv. (total)	5.70	0.21	6.51	0.28	5.74	0.22	5.98	0.14
Tool-use	6.03	0.30	6.82	0.50	5.84	0.31	6.23	0.22
No-tool-use	5.37	0.31	6.21	0.25	5.64	0.30	5.74	0.17
Collab. (total)	7.69	0.18	7.36	0.24	7.38	0.19	7.48	0.12
Tool-use	7.82	0.27	7.47	0.43	7.35	0.29	7.55	0.20
No-tool-use	7.56	0.25	7.25	0.22	7.41	0.26	7.41	0.14
PROCED. (10)								
Indiv. (total)	4.18	0.38	4.43	0.50	3.80	0.39	4.14	0.25
Tool-use	4.50	0.53	5.12	0.89	3.75	0.56	4.46	0.39
No-tool-use	3.86	0.55	3.74	0.45	3.85	0.54	3.82	0.31
Collab. (total)	4.50	0.33	3.80	0.44	4.31	0.35	4.20	0.22
Tool-use	4.68	0.49	2.75	0.77	4.95	0.50	4.13	0.35
No-tool-use	4.32	0.45	4.86	0.40	3.67	0.48	4.28	0.26
SITUAT. (4)								
Indiv. (total)	2.93	0.20	3.17	0.26	3.01	0.18	2.93	0.13
Tool-use	3.41	0.28	3.67	0.46	3.07	0.29	3.38	0.21
No-tool-use	2.46	0.29	2.68	0.24	2.30	0.28	2.47	0.16
Collab. (total)	3.50	0.17	3.71	0.23	3.01	0.18	3.41	0.11
Tool-use	3.55	0.25	3.75	0.40	2.73	0.26	3.34	0.18
No-tool-use	3.46	0.23	3.66	0.21	3.28	0.25	3.47	0.13
OVERALL (26)								
Indiv. (total)	16.31	0.55	17.64	0.83	15.84	0.64	16.60	0.41
Tool-use	17.56	0.88	19.10	1.47	16.58	0.93	17.74	0.65
No-tool-use	15.07	0.91	16.19	0.75	15.11	0.88	15.54	0.51
Collab. (total)	19.32	0.55	18.30	0.72	17.96	0.57	18.53	0.36
Tool-use	19.63	0.81	17.21	1.28	18.26	0.83	18.37	0.58
No-tool-use	19.01	0.74	19.39	0.67	17.66	0.79	18.69	0.42

MANOVAs with setting (individual or collaborative), condition (Conceptual, Arithmetical, or Textual), and tool-use as factors and math grade as covariate were applied to post-test measures.

The outcomes of the analyses showed a difference with regard to *setting*, Wilks' Lambda $F(3,202) = 14.08$, $p < .001$, $\eta_p^2 = 0.17$. Students in the collaborative learning setting showed higher scores with respect to conceptual knowledge ($F(1,204) = 37.56$, $p < .001$, $\eta_p^2 = .16$), situational knowledge ($F(1,204) = 7.65$, $p < .01$, $\eta_p^2 = .04$), and the post-test overall score ($F(1,204) = 12.42$, $p < .001$, $\eta_p^2 = .06$). No difference between individuals and dyads was found with respect to procedural knowledge ($F(1,204) = 0.05$, $p = .83$). Since conceptual knowledge also included intuitive knowledge, a separate MANOVA was performed to establish whether the observed effect on conceptual knowledge concerned intuitive knowledge or the remaining conceptual knowledge items. The analysis showed a students in the collaborative learning setting obtained higher scores with respect to *intuitive* knowledge ($F(1,204) = 66.09$, $p < .001$, $\eta_p^2 = .25$).

No differences were observed for *condition*, Wilks' Lambda $F(8,402) = 1.40$, $p = .19$, and *tool use*, Wilks' Lambda $F(4,201) = 1.47$, $p = .21$.

An interaction was observed between setting and tool-use, Wilks' Lambda $F(3,202) = 3.35$, $p < .05$, $\eta_p^2 = 0.05$. This interaction concerned situational knowledge ($F(1,204) = 9.58$, $p < .01$, $\eta_p^2 = 0.05$) and the post-test overall score ($F(1,204) = 6.08$, $p < .05$, $\eta_p^2 = 0.03$) (see Figure 4-10).



(a) (b)

Figure 4-10. Interactions between setting (individual vs. collaborative learning) and tool use (tool use vs. no tool use) regarding situational knowledge (a) and the post-test overall score (b)

The interaction indicates that regarding situational knowledge and post-test overall scores students in the collaborative setting in general outperformed individual students, but in case individual students constructed a representation their scores equalled those of students in the collaborative setting.

4 DISCUSSION AND CONCLUSION

In this chapter two studies were reported that took place in the same school exactly one year after each other. In one study the affordances and effects of formats of representational tools in an individual learning setting were examined, in the other these were examined in a collaborative learning setting. This allows the data from the collaborative setting to be compared to the data from the individual learning setting. Furthermore, both the study in the individual setting and the study in the collaborative setting were driven by the following questions.

First, are students inclined to use a representation tool and does the format of a tool affect this inclination? Second, does using a representational tool lead to better learning outcomes and are there differences in effectiveness between different formats?

With respect to questions regarding students' inclination to use a representational tool, collaborative and individual students were found to be very like-minded: in both studies, representational tools with a conceptual or textual format were used much more readily (around 50% use) compared to the arithmetical format (around 20% use). Cox (1999) argued that the process of constructing a representation elicits self-explanation effects. Possibly, the arithmetical format is too far removed from the code in which students usually explain the domain to themselves. This would mean that the textual and the conceptual format are more close to the code in which students think (and/or talk) and explain the domain to themselves, or maybe students consider those formats more suited to express their knowledge to the outside world. In both the collaborative setting and the individual setting the formats of the tools did not lead to differential effects on the quality of the constructed representations.

The outcomes of the study suggest that in the collaborative setting the representational tools did not improve learning. Actually, no differences with regard to post-test scores were observed between pairs who used the representational tool to construct a domain representation and pairs who did not use the tool at all, and since their prior knowledge scores were equal, it can be concluded that their learning gains were equal as well. However, a comparison of the scores of collaborative students and those of individual students put the findings in a different perspective. In general, post-test scores in the collaborative setting turned out to be significantly higher than the post-test scores in the individual setting. Only individual students who engaged in constructing a domain representation were found to obtain post-test scores that equalled those of students in the collaborative setting. These findings imply that, with regard to learning outcomes, constructing representations is beneficial for individual students, but in case of collaborative learning constructing representations does not improve learning outcomes. The post-test aimed at measuring three types of knowledge: conceptual knowledge, procedural knowledge, and situational knowledge. Both studies showed that the format of the representational tools did not have differential effects on the different types of knowledge. So, constructing a *conceptual* domain representation for example, did not necessarily lead to enhanced levels of *conceptual* knowledge. However, the data show that individuals constructing representations and collaboration both enhance situational knowledge compared to individuals and pairs respectively that do not construct representations. Furthermore, it was found that students in the collaborative learning setting showed enhanced levels of intuitive knowledge compared to students that learn individually. The observation that collaborative students (regardless of whether or not they constructed a representation) outperformed individuals (even those who did construct a representation), implies that, in this study, intuitive knowledge (a) is enhanced by collaborative learning and (b) constructing representations by individuals can not compensate for the effect of collaboration.

This finding is not consistent with the results of Gijlers and de Jong (submitted), who found that using a concept mapping tool in collaborative learning improved the acquisition of intuitive knowledge compared to collaborative learning without a concept mapping tool. We can only guess what causes this inconsistency of research findings. Indeed, the domain used in both studies differed (mathematics and physics). The same applies to the assignments and measurement instruments, although both studies measured intuitive knowledge. And there are more similarities. For example, both studies applied the same instructional approach, inquiry learning using SimQuest computer simulations, and the participants were equal in terms of educational level and background, age, nationality, and school type.

Since intuitive knowledge is particularly fostered by interpretation and sense-making processes (Gijlers & de Jong, submitted; Reid et al., 2003; Zhang et al., 2004), part of the inconsistency might be explained by the way in which students communicated with their peer. In our study communication between students was on a face-to-face basis, the collaborating students were sitting next to each other, using the same computer terminal. In the study by Gijlers and de Jong, students used a chat-tool to communicate with each other.

Chat communication in collaborative settings is known to put some constraints on communication. For example, in chatting, students tend to be much more succinct, to focus more on technical and organizational issues instead of domain aspects, and to easily jump from topic to topic. This can have positive effects (e.g., brainstorming), but can also be detrimental when the situation requires students to focus on one topic (Anjewierden, Kolloffel, & Hulshof, 2007; Kerr & Murthy, 2004; Strømsø, Grøttum, & Lycke, 2007). In this case, a shared representational tool may not only stimulate interpretation and conclusion activities, but also serve to fill the students' need for an additional channel for communication and reasoning. This is in line with Van Drie et al. (2005) who remarked that (when students communicate via chat) a "representational tool does not only function as a cognitive tool that can elicit elaborative activities, but also as a tool *through which* students communicate" (p. 598).

In a face-to-face setting, communication is not only richer in the sense that it provides both verbal and non-verbal information (e.g., gesturing, nodding, pointing, facial expressions, and intonation of speech), but it also allows students to communicate faster and much more elaborate, which can be crucial in the case of interpretation and sense-making. It could be interesting to explore the relation between mode of communication and the effects on the acquisition of intuitive knowledge in a future study.

For now we conclude that in the domain of combinatorics and probability theory, learning outcomes can be significantly improved by having students learn collaboratively in a face-to-face setting. Our study indicates that if collaborative learning is applied in this domain, representational tools do not seem to have an additional effect on learning outcomes. In the case of individual learning, these tools do help improve learning outcomes. Students who use this type of support, in general obtain learning outcomes equal to those of their collaborating colleagues. Individual students who do not use this support mostly obtain significantly lower learning outcomes. It was also observed that the format of the representational tool can significantly influence the likelihood that students will engage in constructing a representation.

Chapter 5

Discussion and conclusion

1 INTRODUCTION

In the literature we find many references to the proposition that a good match between the type of representation and learning demands can greatly support learning and contribute to enhanced levels of performance and understanding (Ainsworth, 2006; Greeno & Hall, 1997). The studies described in this dissertation focused on the effects of representations in the domain of combinatorics and probability theory, which is a subdomain of mathematics. The overall research question was:

How does representational format facilitate knowledge construction in the domain of combinatorics and probability theory?

The overall research question has been investigated in a step-by-step way. Three studies were conducted, each guided by a different aspect of the overall research question. Study 1 sought to answer the question: *Does representational format influence learning combinatorics and probability theory?* In Study 2 students were provided with representational tools, support tools that can be used to construct external representations of the domain. This study was guided by the question: *What are the effects and perceived affordances of the format of representational tools?* In the last study, Study 3, the question was the same as the question of Study 2, but now this was investigated in a collaborative learning setting. All three studies used Probe, a simulation-based inquiry learning environment on combinatorics and probability theory, and the same knowledge measures. The post-test made a distinction between conceptual, procedural, and situational knowledge. In the next sections the three studies reported in this dissertation will be discussed as well as the relation between representational formats and the three knowledge types. On the basis of these discussions, conclusions will be formulated regarding the overall research question. Furthermore, in line with the aims of the LEMMA research programme (see Chapter 1.3), the findings are translated into practical, evidence-based guidelines for designing learning environments about combinatorics and probability theory for secondary education.

2 EFFECTS OF REPRESENTATIONAL FORMAT ON LEARNING

In Study 1 (presented in Chapter 2) the effects of different representational format were investigated. Students worked with a simulation-based inquiry learning environment in which the formats of the simulations had been manipulated experimentally. Five conditions were compared: three conditions each using a single external representational format and two conditions using multiple representations: (1) tree diagrams; (2) arithmetical equations; (3) text; (4) text + arithmetical equations; or (5) tree diagrams + arithmetical equations. The effects of representational formats were evaluated in terms of effects on learning outcomes and cognitive load.

The findings of the study show that the best learning outcomes were obtained by students who were presented with a combination of text and equations. Their post-test scores with regard to procedural knowledge and their post-test overall scores were significantly higher than those of students presented with tree diagrams or with a combination of tree diagrams and equations. Students presented with tree diagrams reported the highest levels of cognitive load. A combination of tree diagrams and equations was associated with significantly lower levels of cognitive load compared to tree diagrams alone. However, although the cognitive load experienced with the combination of tree diagrams and equations was considerably lower, the learning outcomes were equal to those of students who had been presented with tree diagrams only.

3 AFFORDANCES AND EFFECTS OF REPRESENTATIONAL TOOLS

The aim of the second and the third study was to examine the perceived affordances and effects of representational tools with different formats (conceptual, arithmetical, and textual) on learning outcomes. In the second study, participants learned individually, in the third study participants learned collaboratively. The following questions guided both studies. Do representational formats have differential effects on the likelihood that students use the support and engage in constructing representations? Does format have differential effects on the quality of the representations students construct? Does the construction of a representation of a domain lead to better learning outcomes than not constructing a representation? And, if students construct a representation, does format have differential effects on domain understanding?

3.1 Representational tools in an individual learning setting

The findings of Study 2 (see Chapter 3) show that when students were provided with representational tools with a conceptual or a textual format, respectively 52 and 47 percent of the students used it, whereas 20 percent of students provided with an arithmetical tool used it. For the target population the conceptual or textual formats apparently afford the construction of a representation more than a conventional arithmetical format. This finding to some extent corroborates the observation by Tarr and Lannin (2005) that students initially avoid using conventional ways of representing probabilities, using instead alternative representational forms. It was also observed that formats do not have a differential effect on the quality of the constructed representations.

It was found that the construction of a domain representation significantly improved learning outcomes. Furthermore, it was found that constructing representations, regardless of the format, is associated with significantly higher levels of situational knowledge. The differences could not be attributed to time-on-task. Students in all conditions and regardless whether or not they constructed a domain representation, all spent the same amount of time on their learning task. The formats used to construct domain representations did not have differential effects on conceptual, procedural, or situational knowledge.

3.2 Representational tools in a collaborative learning setting

In Study 3 (see Chapter 4) the set-up was basically the same as in the second study, but now students were working in pairs, doing all activities together except for completing the pre- and post-test. Communication between students was on a face-to-face basis: the collaborating students were sitting next to each other, using the same computer terminal. As was found in the second study, students were much more likely to use representational tools with a conceptual or a textual format (both 45 percent use) than a tool with an arithmetical format (21 percent use). It was also observed that format did not have an effect on the quality of the constructed representations.

The construction of representations did not affect learning outcomes. Actually, pairs who constructed a representation and those who did not obtained equal overall scores on the post-test. Furthermore, no differential effects of formats on conceptual, procedural, or situational knowledge were observed either.

3.3 Comparing representational tools in individual and collaborative settings

With respect to their inclination to use a representational tool, students in both the collaborative and individual setting were found to be very like-minded: representational tools with a conceptual or textual format were used much more readily (around 50% use) compared to the arithmetical format (around 20% use). In both settings the formats of the tools did not lead to differential effects on the quality of the constructed representations. Perhaps, the textual and the conceptual format are more close to the code in which students think (and/or talk) and explain the domain to themselves, or maybe students consider those formats more suited to express their knowledge to the outside world.

With regard to learning outcomes it was found that, in general, post-test scores in the collaborative setting were higher than in the individual setting. Only individual students who engaged in constructing a domain representation in general equalled post-test scores of students in the collaborative setting. It was observed that students learning collaboratively and students constructing domain representations in the individual setting both showed enhanced levels of situational knowledge. This type of knowledge is a prerequisite for going beyond the superficial details of problems in order to recognize the concepts and structures that underlie the problem (e.g., Fuchs et al., 2004). Furthermore, students in the collaborative learning setting showed enhanced levels of intuitive knowledge. The observation that collaborative students (regardless of whether or not they constructed a representation) outperformed individuals (even those who did construct a representation), implied that intuitive knowledge was (a) enhanced by collaborative learning and (b) the activity of constructing representations was not sufficient for individual students to equal the levels of intuitive knowledge of students who worked collaboratively.

4 THE RELATION BETWEEN REPRESENTATIONS AND KNOWLEDGE CONSTRUCTION

It was assumed that understanding of the domain of combinatorics and probability has three components: conceptual knowledge, procedural knowledge, and situational knowledge. In the three studies described in this dissertation it was found that each of these types of knowledge could be influenced.

4.1 Conceptual knowledge

Conceptual knowledge is "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson, Siegler, & Alibali, 2001, p. 364). Enhanced levels of conceptual knowledge were observed when students learned in a collaborative setting. In particular, they showed enhanced levels of intuitive knowledge, that is, they were more accurate at predicting how changes of conceptual parameters (e.g., replacement or no replacement; order important or order not important) would affect the likelihood of certain outcomes in random experiments (drawing marbles from a vase). Such predictions require several concepts and relations between variables to be taken into account simultaneously. People are often not capable of verbalizing the thinking steps on which their predictions are based; therefore the knowledge involved here is called implicit or intuitive (for an extensive discussion, see Swaak & de Jong, 1996). Not much is known about how students acquire this kind of conceptual knowledge, but there are indications that it is particularly fostered by processes of interpretation and sense-making (Gijlers & de Jong, submitted; Reid et al., 2003; Zhang et al., 2004). In the previous section it has been argued that our data indicated that intuitive knowledge was (a) enhanced by collaborative learning and (b) constructing representations did not seem to have an additional effect, at least not on top of collaboration. If the underlying mechanisms are the same, this would suggest that collaboration possibly fosters interpretation and sense-making processes in ways that were not, or to a lesser extent, fostered by learning in the individual setting and/or by engaging in constructing domain representations. However, in collaborative learning the mode of communication may play a role as well. In our study, it did not matter whether or not collaborating pairs constructed a representation; their intuitive knowledge scores were equal (and significantly better than those of individuals) anyway. However, in a study by Gijlers and de Jong (submitted) in which collaborating students did not communicate face-to-face but by means of a chat-tool, it was found that constructing a concept map improved the acquisition of intuitive knowledge compared to collaborative learning without constructing a concept map. This suggests that the mode of communication, face-to-face versus computer-mediated communication might have differential effects on interpretation and sense-making processes. However, more research is needed before solid conclusions can be drawn about the relation between mode of communication and the acquisition of intuitive knowledge.

4.2 Procedural knowledge

Procedural knowledge is the ability to execute steps to solve problems. It was observed that the acquisition of procedural knowledge is influenced by the representational format in which the domain is presented to the students. A combination of a textual and an arithmetical format significantly improved levels of procedural knowledge. The level of cognitive load students experienced with this combination of formats was relatively low. In the case of using a combination of tree diagrams and an arithmetical format, the experienced level of cognitive load was equally low, but this did not result in equal scores with respect to procedural knowledge. This indicates that the used format does influence the construction of procedural knowledge. As discussed in the introductory chapter, the effects of representations found in one domain cannot readily be generalized to other domains (Cheng et al., 2001; Scaife & Rogers, 1996; Zhang, 1997). This raises the question why the combination of text and equations is particularly beneficial in the domain of combinatorics and probability theory. In this domain problem solving requires a set of reasoning steps that are taken in a specific order. The findings suggest that describing these steps in everyday language, followed by an equation concisely repeating these steps in an arithmetical way, particularly facilitates the construction of procedural knowledge.

4.3 Situational knowledge

Situational knowledge (de Jong & Ferguson-Hessler, 1996), enables students to analyze, identify, and classify a problem, to recognize the concepts that underlie the problem, and to decide which operations need to be performed to solve the problem. In Study 1, it was observed that the representational format used to *present* the domain to students does not affect situational knowledge. However, in Study 2 it was found that *constructing* a domain representation, regardless of the format, is associated with significantly higher levels of situational knowledge. This type of knowledge is a prerequisite for going beyond the superficial details of problems in order to recognize the concepts and structures that underlie the problem and to decide which operations are required to solve it (e.g., Fuchs et al., 2004). The format used to construct the domain representation did not directly influence situational knowledge, although it did play an indirect role, since students were found to be more inclined to engage in constructing activities if they could use a conceptual or textual format. In Study 3 it was observed that collaboration is also beneficial for situational knowledge. In that case, it did not make a difference whether students constructed a domain representation, their situational knowledge scores were equal anyway to individual students who had constructed a domain representation.

5 SOME CONSIDERATIONS REGARDING STUDENT CONTROL

Electronic learning environments increasingly offer students many opportunities to exert control over their learning process and the learning environment. Students can choose their own paths through the learning materials, choose the way in which information is presented to them, choose whether or not to use support, and so on. Clarebout and Elen (2006) observed that it is often assumed that providing student control and allowing students to customize their learning environment establishes a ''better'' learning environment, although a clear benefit of student control on learning has not been found yet, except for a positive effect on students' attitudes. Furthermore, they conclude that students often have difficulties making choices that are beneficial for their learning process. This will be illustrated on the basis of two examples: student control with regard to selecting the format in which the learning materials will be presented and student control with regard to using support provided in a learning environment.

5.1 Student control and external representations

If students can select the representational format in which the learning materials will be presented to themselves it becomes important to know how students can, will, or should decide. Without any additional information it can be expected that students will choose on the basis of personal preferences, most likely esthetic preferences (i.e., the format they find most appealing) rather than choices based on insight about their own ways of processing information and learning. Opfermann (2008) observed that students in a hypermedia learning environment who could select the representational format in which they wanted the learning materials to be presented, often did not choose the format that was actually most optimal in terms of their effects on learning outcomes. Although in our studies students could not choose the format of the learning material, we observed similar tendencies. If there was still time left after experiments performed as part of Study 1 (see Chapter 2), a debriefing session was held in which we also asked them for their preference for a representational format. Regarding the five representational formats presented in the study students were all remarkably like-minded. Without exception they found tree diagrams by far the most attractive representation. They stated that if they had had the opportunity to choose, they would have chosen this format. Textual and/or arithmetical representations, which actually led to improved learning performance, were unanimously qualified as boring and tedious. This suggests that what people like (regarding representational formats) does not necessarily lead to better learning outcomes and vice versa.

This seems to be one of the downsides of the current implementations of student control in electronic learning environments. Students have many options to choose from. However, as long as they do not have the knowledge or at least some clues to make informed decisions, they probably fall back on decisions based on superficial features (e.g., choosing the easiest or quickest way) and/or personal preferences (e.g., the appeal of a certain format). As will be discussed in the next section, handing over control to students with regard to the use of support tools provided in the learning environment may cause similar problems.

5.2 Student control and use of support

Learning environments can be enriched with a variety of support tools that have been found to significantly enhance learning outcomes. Yet, in learning environments where students are in control and can autonomously decide on the use of these tools, they can decide *not* to use the tools or even ignore them. In Study 2 and Study 3 for example, the studies in which students were provided with representational tools, it was observed that despite the beneficial effects of constructing representations, considerable numbers of students did not use the tool. There can be several reasons for not using support tools despite their advantageous effects on learning. The tool may not appear useful in the eyes of the students. Furthermore, Clarebout and Elen (2006, in press) argue that metacognitive factors may prevent students from making adequate choices with respect to their learning and the support they need.

Lazonder, Wilhelm, and Ootes (2003) found that the likelihood that students will use a tool is positively correlated with the perceived usefulness of the tool. It should be noted however, that students and designers may have entirely different conceptions of what is useful. Janssen, Erkens, Kirschner, and Kanselaar (in press) point out that what students consider useful "may be very different from what we as educational designers consider useful". Students may be more concerned with 'getting the job done', whereas educational designers may be concerned with fostering the construction of deep and meaningful knowledge. In our Study 2 and Study 3, some students questioned the usefulness of the representational tools. They felt they sufficiently mastered the domain and therefore considered constructing a domain representation not of any added value. Clarebout and Elen (in press) found in this respect a negative relation between students' mastery orientation and tool-use. They suggest this could mean that the more students are mastery oriented the less they see using tools as belonging to the task.

Janssen et al. observed that students' perceptions of the usefulness of support tools often do not correspond to the effectiveness or efficiency of the tools. Clarebout and Elen (2006) add to this that it is reasonable to expect that students will have problems to determine when they need support, what kind of support they need and hence, when the use of tools might be beneficial. On the basis of a review of studies on student control they conclude that students hardly apply monitoring and regulation skills for their learning process or for making adequate choices with respect to their learning. In another study, Clarebout and Elen (in press) examined tool-use in computer-based learning environments. They hypothesized that providing advice to students about the functionality of a support tool in order to help them recognize the opportunity offered by the tool would stimulate students to use tools more frequently. They found that students used tools more frequently and spent more time on using the tools when they first received advice. Furthermore, providing advice neutralized the negative relation between a mastery orientation and tool-use.

5.3 The future of student control

In the previous sections it has been pointed out that providing student control and enabling students to adjust their learning environment does not necessarily establish a ''better'' learning environment, although offering advice, helping students to make informed decisions has shown promising effects, at least in the case of the use of support tools. When students are more aware of their own learning process, their learning (and support) needs, and the beneficial effects of specific external representations and/or support tools, they can more optimally benefit from the control they have over their learning and the learning environment. A disadvantage could be that keeping track of ones own learning process and learning needs and having to consider informed decisions may heavily burden working memory and distract from what it was all about in the first place: learning. The problem started with the freedom electronic learning environments can give students. Perhaps the solution may also lie in electronic learning environments. De Jong (2006) for example suggests that a challenge lies in adapting learning environments to respond to the developing knowledge and skills of students. A cognitive diagnosis, based on the student's interactions with the system, could be used to monitor a student's learning process and developing knowledge and can be used to offer the student tailor-made recommendations, for example with regard to which type of support to use.

For now we have to conclude that there is still much to be explored and examined. In the next section we will return to issues that we already can draw conclusions about.

6 OVERALL CONCLUSION

The main goal of the current research was to find an answer to the question:

How does representational format facilitate knowledge construction processes and how does it influence learning results?

We conclude that in our studies no evidence was found that representational formats affect the construction of conceptual knowledge, neither when used to present information to students, nor when used by students to construct representations. With respect to procedural knowledge it was found that formats directly influence this knowledge type when used to present the domain to students. Finally, in the case of situational knowledge, format used to present the domain to students did not affect this type of knowledge, but format used to construct a representation influenced situational knowledge, say it indirectly.

7 PRACTICAL IMPLICATIONS

The guidelines that flow from the studies reported in this dissertation will be presented below.

Guideline 1: Use a combination of words and equations to present the domain

To present the domain to the student a combination of text and equations is found to lead to the best learning results and relatively low levels of cognitive load. Possibly, this combination of representations fits the strictly sequential nature of problem solving in this domain: in the textual part of the representation the students are taken by the hand as it were, and led step-by-step through an explicit and sequential line of reasoning described in everyday language, followed by an equation concisely repeating these steps in an arithmetical (and also sequential) way.

Guideline 2: Stimulate students to work collaboratively

Collaborative learning helps students to gain better learning outcomes. Our study showed that this even applies to simple forms of collaborative learning. For example, in our study no collaboration scripts were used and pairs were not matched in advance by the experimenter. Students were allowed to choose their partner themselves. Communication between students was on a face-to-face basis: the collaborating students were sitting next to each other, using the same computer terminal. Having students work together is found to enhance conceptual (intuitive) knowledge, situational knowledge, and overall post-test scores.

Guideline 3: Have individuals construct a concept map or write a summary

If students work individually, then it is worthwhile to have them construct an external representation. For the target population conceptual or textual formats apparently afford the construction of representation more than a conventional arithmetical format. Constructing a domain representation is associated with higher levels of situational knowledge and overall post-test scores.

These guidelines not only show how conceptual, procedural, and situational knowledge can be improved, but each of them has been found to enhance learning outcomes in general as well. These guidelines complement each other: the best learning results can therefore to be expected if guidelines are implemented together.

Summary

1 INTRODUCTION

External representations can have many formats (e.g., pictures, diagrams, texts, graphs, and tables) A good match between the representational format and learning demands can greatly support learning and contribute to enhanced levels of performance and understanding (Ainsworth, 2006; Greeno & Hall, 1997). Representations can help students to select, organize, and integrate information into meaningful and coherent internal representations, being it by communicating information to students in clear and understandable ways, or by serving as a means through which students express, refine, and communicate their understanding. Unfortunately, a clear-cut recipe for which representational format to use when does not exist. Moreover, some researchers argue that the effects of representations found in one domain cannot readily be generalized to other domains (Cheng et al., 2001; Scaife & Rogers, 1996; Zhang, 1997). The studies described in this dissertation focus on the effects of representations in the domain of combinatorics and probability theory, which is a subdomain of mathematics. This domain of mathematics is hard to grasp for many students. Some of the students' difficulties with mathematics are caused by the abstract and formal nature of arithmetical representations which do not explicitly show the underlying principles or concepts. Most students tend to view mathematical symbols (e.g., multiplication signs) purely as indicators of which operations need to be performed on adjacent numbers, rather than reflections of principles and concepts underlying these procedures (Atkinson et al., 2003; Cheng, 1999; Greenes, 1995; Nathan et al., 1992; Niemi, 1996; Ohlsson & Rees, 1991). Complicating factor is that combinatorial and probability problems and ideas often appear to conflict with conceptions and intuitions people have (Batanero & Sanchez, 2005; Fischbein, 1975; Garfield & Ahlgren, 1988; Greer, 2001; Kapadia, 1985). Like most problems in mathematics, problems involving combinatorics and probability theory require students to go beyond the superficial details in order to recognize the concepts and structures that underlie the problem and to decide which operations need to be performed to solve it (e.g., Fuchs et al., 2004).

2 RESEARCH QUESTION

The overall research question of the project was: how does representational format facilitate knowledge construction processes and how does this influence learning in the domain of combinatorics and probability theory? The overall research question has been investigated in a step-by-step way. Three studies were conducted, each guided by a different aspect of the overall research question. Study 1 sought to answer the question: *Does representational format influence learning combinatorics?* In Study 2 students were provided with representational tools, support tools that can used to construct external representations. Three types of representational tools were compared, namely tools that could by used to construct a conceptual, an arithmetical, or a textual domain representation. This study was guided by the question: *What are the effects and perceived affordances of the format of representational tools?* In the last study, Study 3, the question was: *What are the effects and perceived affordances of the format of representational tools in a collaborative learning setting?* All three studies used basically the same simulation-based inquiry learning environment on combinatorics and probability theory and the same knowledge measures.

2.1 Instruction and learning environment

The instructional approach used in the reported studies was simulation-based inquiry learning (de Jong, 2005, 2006). In inquiry learning, the focus of instruction is primary on the induction of concepts and principles of a domain (Swaak & de Jong, 1996). Students inquire the properties of the given domain (de Jong & van Joolingen, 1998; van Joolingen, 1993; van Joolingen & de Jong, 1997). Computer-based simulation is a technology that is particularly suited for inquiry learning. Computer-based simulations contain a model of a system or a process. By manipulating the input variables and observing the resulting changes in output values the student is enabled to induce the concepts and principles underlying the model (de Jong & van Joolingen, 1998). All studies used Probe, a simulation-based inquiry learning environment about combinatorics and probability theory. Probe was created with SimQuest authoring software (van Joolingen & de Jong, 2003). Probe consisted of several sections, each treating a different problem category in the domain of combinatorics and probability theory. All sections had the same structure and a series of assignments (both open-ended and multiple-choice items). Depending on the goals of the subsequent studies Probe could be adjusted. In Study 1 five versions of Probe were used that were all identical except for the representational format of the simulations, In Study 2 and 3 Probe was equipped with representational tools.

2.2 Knowledge measures

The knowledge tests, in particular the post-test, were based on the assumption that understanding in the domain of combinatorics and probability has three aspects: conceptual knowledge, procedural knowledge, and situational knowledge. *Conceptual knowledge* is "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson et al., 2001, p. 364). *Procedural knowledge* is "the ability to execute action sequences to solve problems" (Rittle-Johnson et al., 2001, p. 346). *Situational knowledge* (de Jong & Ferguson-Hessler, 1996) enables students to analyze, identify, and classify a problem, to recognize the concepts that underlie the problem, and to decide which operations need to be performed to solve the problem. All three studies employed a between-subjects pre-test post-test design.

3 EMPIRICAL STUDIES

3.1 Study 1

The first study investigated which representations help students best to acquire domain knowledge. In this first study students worked individually. In total, 123 students participated in the study. The experiment employed a between-subjects pre-test post-test design in which the representational format used to present the domain was manipulated. Five conditions were compared to each other: three conditions each using a single external representational format (Diagrammatical, Arithmetical, or Textual), and two conditions using combinations of single representational formats (Textual + Arithmetical or Diagram + Arithmetical). The effects of representational formats were evaluated in terms of effects on knowledge construction and efficiency. The main finding of the study is that learning from a format that combines text and arithmetic was most beneficial, in particular with regard to procedural knowledge. Diagrams were found to negatively affect learning and to increase cognitive load. Combining diagrams with arithmetical representations reduced cognitive load, but did not improve learning outcomes.

3.2 Study 2

The aim of the second study was to examine the affordances and effects of representational tools with different formats on learning outcomes. Representational tools are support tools which students can use to construct external representations. Format of the tool is the language of expression that is used (conceptual, arithmetical, or textual). The following two questions guided the study. First, are students inclined to use a representation tool and does format affect this inclination? Second, does using a representational tool lead to better learning outcomes and are there differences in effectiveness between different formats? A between-subject pre-test-post-test design was applied with 127 secondary education students randomly distributed over four conditions. Participants in three experimental conditions were provided with a representational tool that could be used to construct a domain representation. The experimental manipulation concerned the format of the tool (conceptual, arithmetical, or textual). The *conceptual representational tool* could be used to create a concept map of the domain. In the *arithmetical representational tool* students could use variable names, numerical data, equations, and formulas to express their knowledge. Finally, the *textual representational tool* resembled word processing software, allowing textual input. Participants in a control condition did not have access to a representational tool, but for the rest their learning environment and tasks were identical to those of students in the experimental conditions. It was found that the construction of a domain representation significantly improved learning outcomes. It was also found that the format of the representational tool significantly influenced the likelihood that students would engage in constructing a representation: when provided with a conceptual or a textual tool, respectively 52 and 47 percent of the students used it, whereas only 20 percent of the students provided with the arithmetical tool used it.

3.3 Study 3

In the third study the collaboration aspect was introduced. Basically, the set-up was the same as in the second study, but now students were learning collaboratively. The representational tool was a shared representational tool that again was pre-structured in either a conceptual, arithmetical, or textual way. The aim of the study was to examine the affordances and effects of representational tools with different formats on learning outcomes in a collaborative learning setting. The following two questions guided the study. First, are students inclined to use a representation tool and are there differences in attractiveness between formats? Second, does using a representational tool lead to better learning outcomes and are there differences in effectiveness between different formats? In a between-subjects pre-test post-test design, data were collected of 61 pairs, who were randomly distributed over three conditions (conceptual, arithmetical and textual format). It was found that the construction of a domain representation did not improve learning outcomes in this collaborative learning setting: students who used the support tool to construct a domain representation obtained post-test scores equal to those of students who had not used the tool. In this study it was also found that the format of the representational tool could significantly influence the likelihood that students would engage in constructing a representation: when provided with a conceptual or a textual tool, 45 percent of the pairs used it, whereas only 21 percent of the pairs provided with the arithmetical tool used it.

3.4 Comparing Study 2 and 3

With respect to students' inclination to use a representation tool, collaborative and individual students responded very similar to the tools: in both studies, students were much more apt to use representational tools with a conceptual or textual format (around 50% use) compared to the arithmetical format (around 20% use). In general, post-test scores in the collaborative setting turned out to be significantly higher than the post-test scores in the individual setting. Only individual students who engaged in constructing a domain representation were found to obtain post-test scores that equalled those of students in the collaborative setting. These findings imply that, with regard to learning outcomes, constructing representations is beneficial for individual students, but in case of collaborative learning constructing representations does not improve learning outcomes. Both studies showed that the format of the representational tools did not have differential effects on the different types of knowledge. So, constructing an *arithmetical* domain representation for example, did not necessarily lead to enhanced levels of *procedural* knowledge. However, the data show that individuals constructing representations and collaboration both enhance situational knowledge. Furthermore, it was found that students in the collaborative learning setting showed enhanced levels of intuitive conceptual knowledge. The observation that collaborative students (regardless of whether or not they constructed a representation) outperformed individuals (even those who did construct a representation), implies that, in this study, intuitive conceptual knowledge (a) is enhanced by collaborative learning and (b) is not related to constructing representations.

4 GENERAL CONCLUSION

The overall research question of the project was: how does representational format facilitate knowledge construction processes and how does this influence learning? The findings of the three studies presented before show that the acquisition of procedural knowledge can be influenced directly by representational format. Conceptual and situational knowledge appear not to be influenced by the format in which the learning material is presented. However, it was found that the acquisition of situational knowledge can be facilitated by constructing representations and/or by learning collaboratively. Higher levels of conceptual knowledge are to be expected in case students learn collaboratively.

The research findings were translated into three practical, evidence-based guidelines for designing learning environments about combinatorics and probability theory for secondary education.

Guideline 1: Use a combination of words and equations to present the domain

Using a combination of text and equations to present the domain to students is found to lead to the best learning results (in particular procedural knowledge) and relatively low levels of cognitive load.

Guideline 2: Stimulate students to work collaboratively

Collaborative learning helps students to gain better learning outcomes. Our study showed that this even applies to simple forms of collaborative learning. It was found to enhance conceptual (intuitive) knowledge, situational knowledge, and overall post-test scores.

Guideline 3: Have individuals construct a concept map or write a summary

If students work individually, then it is worthwhile to have them construct an external representation. For the target population conceptual or textual formats apparently afford the construction of representation more than a conventional arithmetical format. Constructing a domain representation is associated with higher levels of situational knowledge and overall post-test scores.

The best learning results are to be expected if guidelines are implemented together, since they can complement each other.

Samenvatting

1 INTRODUCTIE

Externe representaties zijn er in vele typen (bijvoorbeeld plaatjes, diagrammen, teksten, grafieken en tabellen). Een goede aansluiting tussen het representatietype en de leertaak kan een belangrijke ondersteuning vormen van het leerproces en bijdragen aan verbetering van leerprestaties en begripsvorming (Ainsworth, 2006; Greeno & Hall, 1997). Representaties kunnen leerlingen helpen om informatie te selecteren, te organiseren en te integreren in betekenisvolle en coherente interne representaties, hetzij door informatie op duidelijke en inzichtelijke wijze aan leerlingen te presenteren, hetzij door als media te dienen door middel waarvan leerlingen hun kennis tot uitdrukking kunnen brengen, kunnen verfijnen en kunnen communiceren. Helaas bestaat er geen kant-en-klaar recept voor welk representatietype wanneer gebruikt moet worden. Bovendien stellen een aantal onderzoekers dat de effecten van representaties die in een bepaald kennisdomein gevonden worden, vaak niet of nauwelijks gegeneraliseerd kunnen worden (Cheng et al., 2001; Scaife & Rogers, 1996; Zhang, 1997). De studies die in dit proefschrift gerapporteerd worden, richten zich op de effecten van representaties in het kennisdomein combinatoriek en kansrekening, een subdomein van wiskunde. Dit domein binnen wiskunde is voor veel leerlingen moeilijk te begrijpen. Een deel van de moeite die leerlingen met wiskunde hebben wordt veroorzaakt door de abstracte en formele aard van wiskundige representaties, die vaak de onderliggende principes en concepten niet expliciet weergeven. Veel leerlingen hebben de neiging om wiskundige symbolen, bijvoorbeeld het vermenigvuldigingsteken, te beschouwen als indicaties van de bewerkingen die ze met de aangrenzende getallen uit moeten voeren, in plaats van dat ze de symbolen zien als een uitdrukking van principes of concepten (Atkinson et al., 2003; Cheng, 1999; Greenes, 1995; Nathan et al., 1992; Niemi, 1996; Ohlsson & Rees, 1991). Een complicerende factor is dat combinatoriek en kansrekening vaak strijdig lijken met de ideeën en intuïties van mensen (Batanero & Sanchez, 2005; Fischbein, 1975; Garfield & Ahlgren, 1988; Greer, 2001; Kapadia, 1985). Zoals het geval is bij de meeste wiskundige problemen, vereist het oplossen van combinatorische en waarschijnlijkheidsvraagstukken dat leerlingen verder kijken dan de oppervlakkige kenmerken van het probleem. Om te beslissen welke procedures nodig zijn om het vraagstuk op te lossen, moeten de onderliggende concepten en structuren herkend worden (zie bijv. Fuchs et al., 2004).

2 ONDERZOEKSVRAAG

De hoofdvraag van dit project was: *Hoe bevordert representatietype het construeren van kennis door leerlingen en welke invloed heeft dit op het leren van combinatoriek en kansrekening?* Om deze vraag te beantwoorden zijn drie studies verricht, die zich elk op een verschillend aspect van de hoofdvraag richtten. In Studie 1 is gezocht naar het antwoord op de vraag: *Beïnvloedt het representatietype waarmee het domein gepresenteerd wordt het leren van combinatoriek en kansrekening?* In Studie 2 kregen leerlingen de beschikking over een zogenaamde *representational tool*, dat wil zeggen: een elektronisch hulpmiddel waarmee ze zelf externe representaties konden construeren. Drie typen representational tools werden onderzocht: namelijk tools waarmee een conceptuele, een wiskundige of een tekstuele representatie van het domein geconstrueerd kon worden. In deze studie stond de vraag centraal: *Wat zijn de effecten van de verschillende typen representational tools op* *leren en welke functionaliteit hebben ze in de ogen van leerlingen ("perceived affordances")?* In de laatste studie, Studie 3, werd dezelfde vraag onderzocht als in Studie 2, maar dan in een situatie waarin leerlingen samenwerkten tijdens het leren. Alle studies maakten gebruik van dezelfde leeromgeving voor onderzoekend leren met behulp van computersimulaties, van dezelfde materialen (introductie, voorbeelden, opdrachten) en van dezelfde kennistests (in Studie 2 en 3 werd echter een verkorte versie van de natoets gebruikt).

2.1 Instructie en leeromgeving

De instructiebenadering die in de gerapporteerde studies is gebruikt, is onderzoekend leren met behulp van computersimulaties (de Jong, 2005, 2006). In onderzoekend leren richt instructie zich primair op het onderzoeken van domeinconcepten en –principes (de Jong & van Joolingen, 1998; Swaak & de Jong, 1996; van Joolingen, 1993; van Joolingen & de Jong, 1997). Computersimulaties zijn bij uitstek geschikt voor onderzoekend leren. Simulaties bevatten een model van een systeem of een proces. Door in de simulatie de waarden van bepaalde variabelen te veranderen en daarbij te letten op de effecten van deze veranderingen op andere variabelen, kunnen leerlingen de concepten en principes achterhalen die aan het model ten grondslag liggen (de Jong & van Joolingen, 1998). In alle studies werd gebruik gemaakt van Probe, een elektronische leeromgeving ingericht voor onderzoekend leren met computersimulaties in het domein van combinatoriek en kansrekening. Probe bestond uit verschillende modules, waarbij elke module een bepaald onderdeel van combinatoriek en kansrekening behandelde. Alle modules hadden dezelfde structuur. De leerlingen kregen een aantal vragen en opdrachten gepresenteerd die ze met behulp van de in Probe geïntegreerde simulaties konden beantwoorden. Al naar gelang de doelstellingen van de onderzoeken werd Probe aangepast. In Studie 1 werden vijf versies van Probe gebruik die alle identiek waren, behalve ten aanzien van het representatietype dat gebruikt werd om variabelen in de simulaties weer te geven. In Studie 2 en 3 werd Probe uitgerust met representational tools.

2.2 Kennistests

De kennistests, in het bijzonder de natoets, waren gebaseerd op de aanname dat domeinkennis ten aanzien van combinatoriek en kansrekening drie componenten heeft: conceptuele kennis, procedurele kennis en situationele kennis. *Conceptuele kennis* is impliciete of expliciete kennis van de belangrijkste principes en relaties tussen concepten binnen het kennisdomein (Rittle-Johnson et al., 2001). *Procedurele kennis* is het vermogen om procedures uit te voeren die leiden tot het oplossen van het probleem (Rittle-Johnson et al., 2001). *Situationele kennis* (de Jong & Ferguson-Hessler, 1996) stelt leerlingen in staat om problemen te identificeren, te analyseren en te classificeren, om de onderliggende concepten te herkennen en te beslissen welke procedures uitgevoerd moeten worden om het probleem op te lossen. Alle drie de studies maakten gebruik van een *between-subjects,* voortoets-natoets onderzoeksopzet.

3 EMPIRISCHE STUDIES

3.1 Studie 1

In de eerste studie werd onderzocht welk representatietype leerlingen het best helpt om domeinkennis te verwerven. Leerlingen werkten zelfstandig. In totaal namen 123 Vwo-leerlingen deel aan het onderzoek. In het onderzoek werden vijf representatietypen onderzocht waarmee variabelen in de simulaties werden weergegeven. De effecten van representatietypen werd onderzocht door vijf experimentele condities met elkaar te vergelijken: drie condities met een enkelvoudig representatietype (Boomdiagram, Vergelijking of Tekst) en twee condities met een combinatie van twee representatietypen (Tekst + Vergelijking of Boomdiagram + Vergelijking). De representatietypen werden beoordeeld op grond van hun effecten op kennisconstructie en mentale belasting. De belangrijkste bevinding van deze studie was dat het leren met de combinatie Tekst + Vergelijking leidde tot de beste leerresultaten, in het bijzonder procedurele kennis. Boomdiagrammen bleken een negatieve invloed te hebben op leerresultaten en te leiden tot een grotere mentale belasting. De combinatie Boomdiagram + Vergelijking reduceerde de ervaren mentale belasting, maar leidde niet tot betere leerprestaties.

3.2 Studie 2

Het doel van de tweede studie was om verschillende typen representational tools te vergelijken in termen van hun effecten op leerprestaties en hun functionaliteit in de ogen van leerlingen (*perceived affordances*). Een representational tool is een hulpmiddel waarmee leerlingen zelf representaties van het kennisdomein kunnen construeren. Drie typen representational tools werden onderzocht: tools waarmee conceptuele, wiskundige of tekstuele representaties van het domein geconstrueerd kunnen worden. In een conceptuele representatie (ook wel "*concept map*" of "begrippenkaart" genoemd), kunnen leerlingen een schematisch overzicht maken van de belangrijkste domeinconcepten en hun onderlinge relaties. Doorgaans wordt elk concept in een of enkele woorden in een cirkel geschreven en met behulp van pijlen tussen de verschillende cirkels worden de onderlinge relaties (en de richtingen van de relaties) weergegeven. In een wiskundige representatie, worden de belangrijkste domeinkenmerken in wiskundige vorm genoteerd (bijv. vergelijkingen en formules). Een tekstuele representatie tenslotte, is een tekstuele verhandeling over het domein, vergelijkbaar met een samenvatting. De volgende vragen vormden de basis van de studie. Ten eerste, zijn leerlingen geneigd een representational tool te gebruiken en speelt het type representational tool daarbij een rol? Ten tweede, leidt het gebruik van een representational tool tot betere leerresultaten en leiden verschillende typen representational tools tot verschillende leerresultaten? Aan het onderzoek namen 133 Vwo-leerlingen deel die willekeurig over vier condities verdeeld werden. Deelnemers in drie experimentele condities hadden de beschikking over een (conceptuele, wiskundige of tekstuele) representational tool. Leerlingen in de controleconditie hadden niet de beschikking over een representational tool, maar verder was hun leertaak identiek aan die van leerlingen in de experimentele condities. Het onderzoek toonde aan dat het construeren van een domeinrepresentatie leidde tot een significante verbetering van leerresultaten. Er werd ook gevonden dat de verschillende typen representational tools geen direct effect op leerprestaties hebben, maar dat ze wel een significante invloed hebben op de waarschijnlijkheid dat leerlingen overgaan tot het construeren van een representatie: het conceptuele en tekstuele type werden door respectievelijk 52 en 47 procent van de leerlingen gebruikt, terwijl 20 procent van de leerlingen die de beschikking hadden over het wiskundige type er gebruik van maakten.

3.3 Studie 3

In de derde studie werd samenwerkend leren geïntroduceerd. In principe was de onderzoeksopzet gelijk aan de opzet van Studie 2, maar nu werkten de leerlingen in tweetallen aan de leertaak. Dezelfde typen representational tools (conceptueel, wiskundig of tekstueel) werden gebruikt. De vraagstelling die aan deze studie ten grondslag lag was dezelfde als in Studie 2. Ten eerste, zijn leerlingen geneigd een representational tool te gebruiken en speelt het type representational tool daarbij een rol? Ten tweede, leidt het gebruik van een representational tool tot betere leerresultaten en leiden verschillende typen representational tools tot verschillende leerresultaten? Er werd data verzameld van 61 tweetallen (Vwo-leerlingen) die willekeurig over de drie condities waren verdeeld. Uit het onderzoek kwam naar voren dat het construeren van een domeinrepresentatie door tweetallen niet leidt tot verbetering van leerprestaties: de leerresultaten van tweetallen die een representatie construeerden en de resultaten van tweetallen die dat niet deden, waren gelijk. Net als in Studie 2 werd gevonden dat het type representational tool een significante invloed heeft op de waarschijnlijkheid dat leerlingen overgaan tot het construeren van een representatie: het conceptuele en tekstuele type werden beide door 45 procent van de tweetallen gebruikt, terwijl van 21 procent van de tweetallen die de beschikking hadden over het wiskundige type er gebruik van maakten.

3.4 Vergelijking van Studie 2 en 3

Ten aanzien van het gebruik van representational tools waren de individuele leerlingen in Studie 2 en de tweetallen in Studie 3 gelijkgestemd: in beide studies waren leerlingen meer geneigd om een conceptuele of tekstuele representatie te maken (rond 50 procent) dan een wiskundige representatie (ongeveer 20 procent). Verder bleek dat de leerresultaten van leerlingen die samen hadden gewerkt, gemiddeld gezien beter waren dan die van leerlingen die alleen hadden gewerkt. Alleen als individuele leerlingen een representatie van het domein hadden geconstrueerd, waren hun leerresultaten vergelijkbaar met die van samenwerkende leerlingen. Deze bevindingen impliceren dat het construeren van een domeinrepresentatie leidt tot betere leerprestaties bij individuen, maar niet bij samenwerkende leerlingen. Beide studies laten zien dat het type representational tool geen differentiële effecten op verschillende kennistypen heeft. Bijvoorbeeld, het construeren van een conceptuele domeinrepresentatie leidt niet tot meer conceptuele kennis. Er is echter wel gevonden dat zowel het construeren van een domeinrepresentatie door individuele leerlingen, als het samenwerken van leerlingen leidt tot een toename van situationele kennis. Daarnaast werd gevonden dat samenwerkend leren leidt tot een toename van intuïtieve kennis. Aangezien leerlingen die in tweetallen hebben gewerkt structureel beter scoren op dit type kennis vergeleken met individuen, ook degenen die een domeinrepresentatie geconstrueerd hebben, impliceert dit dat in deze studie intuïtieve kennis (a) toeneemt onder invloed van samenwerking en (b) het construeren van domeinrepresentaties geen effect, althans geen effect bovenop het effect van samenwerken heeft en daarnaast onvoldoende is om de intuïtieve kennis van individuen op te trekken naar het niveau van leerlingen die samen hadden geleerd.

4 ALGEMENE CONCLUSIE

De hoofdvraag van dit project was: Hoe bevordert representatietype kennisconstructie en welke invloed heeft dit op leren? De resultaten van de drie studies laten zien dat het verwerven van procedurele kennis direct beïnvloed wordt door representatietype. Conceptuele en situationele kennis lijken niet te worden beïnvloed door het type representatie dat gebruikt wordt om het lesmateriaal te presenteren. Echter, het onderzoek laat ook zien dat het verwerven van situationele kennis bevorderd kan worden door het construeren van domeinrepresentaties door leerlingen en/of door samenwerkend leren. Een toename van conceptuele kennis in dit domein valt te verwachten van het laten samenwerken van leerlingen.

Op basis van de onderzoeksresultaten zijn drie richtlijnen geformuleerd voor het ontwerpen van leeromgevingen voor combinatoriek- en kansrekeninginstructie in het middelbaar onderwijs:

Richtlijn 1: Gebruik een combinatie van woorden en vergelijkingen om het domein te presenteren
Uit het onderzoek komt naar voren dat het presenteren van de leerstof door middel van een combinatie van tekst en vergelijkingen leidde tot de beste leerresultaten (in het bijzonder procedurele kennis) en relatief weinig mentale belasting.

Richtlijn 2: Stimuleer leerlingen om samen te werken
De onderzoeksresultaten laten zien dat samenwerking leidt tot betere leerresultaten vergeleken met individueel leren. Bovendien laat het onderzoek zien dat dit al met eenvoudige middelen te bereiken is. Leerlingen in tweetallen naast elkaar laten zitten en gezamenlijk de leerstof laten doorlopen bleek al voldoende om significant betere leerresultaten te boeken. Samenwerking bleek in het bijzonder intuïtieve en situationele kennis te bevorderen.

Richtlijn 3: Laat individuele leerlingen een concept map construeren of een samenvatting schrijven
Als leerlingen alleen werken dan is het de moeite waard om ze een representatie van het domein te laten construeren. Voor de doelgroep is een concept map of een tekstuele representatie het meest toegankelijk. Het construeren van een domein representatie leidt tot betere leerresultaten (in het bijzonder situationele kennis).

De richtlijnen laten niet alleen zien hoe conceptuele, procedurele en situationele kennis verbeterd kunnen worden, maar elk leidt ook tot een verbetering van de algehele leerresultaten. De richtlijnen vullen elkaar aan, daarom kunnen de beste leerresultaten verwacht worden als richtlijnen gecombineerd worden.

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Appendix

	REPRESENTED?	CONCEPTUAL TOOL	ARITHMETICAL TOOL	TEXTUAL TOOL	PNT
A	The concept of "Replacement"	-Literally, or descriptive <u>Examples</u>: *-"Replacement"* *-"Category 1: without replacement; order important"* *-...[Runners, BK]... then you have to do 1/7 x 1/6 x 1/5 because each time there is one runner fewer"*	Two formulas or calculations in which "replacement" varies <u>Examples</u>: *-"(1/n) x (1/n) x (1/n) = P* *(1/n) x (1/(n-1)) x (1/(n-2))= P"* *-"1/5 x 1/4 x 1/3* *1/5 x 1/5 x 1/5"* *-"p = 1/10 x 1/10 x 1/10* *p = 1/5 x 1/4 x 1/3"*	-Literally, or descriptive <u>Examples</u>: *-"Replacement"* *-"Category 1: without replacement; order important"* *-"...If there are 7 runners, then the chance is 1 out of 7 (1/7), if that runner passes the finish, then there are 6 runners left, then there is a chance of 1 out of 6 (1/6), and so on.*	1
B	The concept of "Order"	-Literally, or descriptive <u>Examples</u>: *-"Order"* *-"Category 1: without replacement; order important"* *-"...If there are 7 runners and you predict the top 3 without specifying the positions of specific runners in the top 3..."*	Two formulas or calculations in which "order" varies <u>Examples</u>: *-"(1/n) x (1/n) x (1/n)* *(k/n) x ((k-1)/n) x ((k-2)/n)"* *-"1/5 x 1/4 x 1/3* *3/5 x 2/4 x 1/3"*	-Literally, or descriptive <u>Examples</u>: *-"Order"* *-"Category 1: without replacement; order important"* *-"...At a game of Bingo, order is not important"*	1
C	Calculation	-Formal, literally, descriptive, or a concrete calculation <u>Examples</u>: *-p = acceptable outcomes/ possible outcomes* *- 1/5 x 1/4 x 1/3* *-... when you also bet ont he order in which the marbles will be selected, your chance is: 1/5 and 1/4 is 1/20..."*	Formal (formula) or a concrete calculation <u>Examples</u>: *-"(1/n) x (1/n) x (1/n)"* *-"1/5 x 1/4 x 1/3"*	-Formal, literally, descriptive, or a concrete calculation <u>Examples</u>: *-p = acceptable outcomes/ possible outcomes* *- 1/5 x 1/4 x 1/3* *-... when you also bet ont he order in which the marbles will be selected, your chance is: 1/5 and 1/4 is 1/20..."*	1

	REPRESENTED?	CONCEPTUAL TOOL	ARITHMETICAL TOOL	TEXTUAL TOOL	PNT
D	Probability	-Literal reference to the term "probability"/p, or a description of the concept -Expression of a concrete probability (e.g. a fraction), but then it need to be made clear in the context (e.g. by a calculation) where the probability comes from *<u>Examples</u>:* *-"In order to calculate 'p' the chances need to be multiplied."* *-p= 1/5 x 1/4 x 1/3* *-"...In that case [student refers to a situation outlined earlier], the probability is 1/10"*	-Literal reference to the term "p" -Expression of the outcome of a calculation *<u>Examples</u>:* *-"p = (1/n) x (1/n) x (1/n)"* *-"p = 1/5 x 1/4 x 1/3"* *-"1/5 x 1/4 x 1/3 = 1/60"*	-Literal reference to the term "probability"/p, or a description of the concept -Expression of a concrete probability (e.g. a fraction), but then it need to be made clear in the context (e.g. by a calculation) where the probability comes from *<u>Examples</u>:* *-"In order to calculate 'p' the chances need to be multiplied."* *-p= 1/5 x 1/4 x 1/3* *-"...In that case [student refers to a situation outlined earlier], the probability is 1/10"*	1
E	Effect of *n* on probability	-Descriptive or on basis of calculations showing the effect (in the latter case, *k* needs to be constant) *<u>Examples</u>:* *-"fewer options = higher chance"* *-"If fewer runners attend the race, the chance your prediction is correct will increase"*	A formula or a series of calculations showing the effect (in the latter case, *k* needs to be constant) *<u>Examples</u>:* *-"(1/n) x (1/n) x (1/n)=1/n³"* *-"1/5 x 1/4 x 1/3 = 1/60* *1/6 x 1/5 x 1/4 = 1/120"*	-Descriptive or on basis of calculations showing the effect (in the latter case, *k* needs to be constant) *<u>Examples</u>:* *-"If the number of elements you can choose from increases, the chance will be smaller that you will select a specific element"* *-"If fewer runners attend the race, the chance your prediction is correct will increase"*	1

	REPRESENTED?	CONCEPTUAL TOOL	ARITHMETICAL TOOL	TEXTUAL TOOL	PNT
F	Effect of *k* on probability	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* needs to be constant) <u>Examples</u>: -*"with 1 choice →1/possible outcomes; with more choices →number of choices/possible outcomes"* -*"If you only predict who will win the race and not the top 3, then the chance is greater that your prediction will be correct"*	A formula or a series of calculations showing the effect (in the latter case, *k* needs to be constant) <u>Examples</u>: -"$(1/n) \times (1/n) = 1/n^2$ $(1/n) \times (1/n) \times (1/n) = 1/n^3$" -"$1/5 \times 1/4 = 1/20$ $1/5 \times 1/4 \times 1/3 = 1/60$"	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* needs to be constant) <u>Examples</u>: -*"When your prediction is less elaborate, the probability that your prediction will be correct increases"* -*"If you only predict who will win the race and not the top 3, then the chance is greater that your prediction will be correct"*	1
G	Effect of replacement on probability	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* and *k* need to be constant) <u>Examples</u>: -*"If it is a matter of replacement, your chances will decrease"* -*"...if you have 10 different cell phones and you need to select one, your chance will be 1 out of 10, if you put the phone back your chance will be 1 out of 10 again, but if you leave it out your chance will increase that you will select the next phone as predicted"*	A series of formulas or calculations showing the effect, but the outcome (*p*) needs to be represented as well and *n* and *k* need to be constant <u>Examples</u>: -"$(1/n) \times (1/n) = 1/n^2$ $(1/n) \times (1/(n-1)) = 1/(n^2-n)$" -"$1/5 \times 1/4 \times 1/3 = 1/60$ $1/5 \times 1/5 \times 1/5 = 1/125$"	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* and *k* need to be constant) <u>Examples</u>: -*"If it is a matter of replacement, your chances will decrease"* -*"...if you have 10 different cell phones and you need to select one, your chance will be 1 out of 10, if you put the phone back your chance will be 1 out of 10 again, but if you leave it out your chance will increase that you will select the next phone as predicted"*	1

	REPRESENTED?	CONCEPTUAL TOOL	ARITHMETICAL TOOL	TEXTUAL TOOL	PNT
H	Effect of order on probability	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* and *k* need to be constant) <u>*Examples*</u>*:* *-" If order is important, the chance your prediction will be right will decrease"* *-"...If there are 7 runners and you predict the top 3, then the probability is 1/7 x 1/6 x 1/5 = 1/210, but without specifying the positions of specific runners in the top 3 the probability is 3/7 x 2/6 x 1/5 = 6/210..."*	A series of formulas or calculations showing the effect, but the outcome (*p*) needs to be represented as well and *n* and *k* need to be constant <u>*Examples*</u>*:* -"$(1/n) \times (1/n) = 1/n^2$ $(k/n) \times ((k-1)/n) = (k^2-k)/n^2$" -"$1/5 \times 1/4 \times 1/3 = 1/60$ $3/5 \times 2/4 \times 1/3 = 6/60$"	-Descriptive or on basis of calculations showing the effect (in the latter case, *n* and *k* need to be constant) <u>*Examples*</u>*:* *-" If order is important, the chance your prediction will be right will decrease"* *-"...If there are 7 runners and you predict the top 3, then the probability is 1/7 x 1/6 x 1/5 = 1/210, but without specifying the positions of specific runners in the top 3 the probability is 3/7 x 2/6 x 1/5 = 6/210..."*	1
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